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Computational Geometry — Homework 2 — HS13

Exercise 1 (10 Points)

Let $P = \{p_1, p_2, \ldots, p_n\}$ be a set of points in the plane (not all of them on the same line). Prove that there is a triangulation of the vertices of the convex hull $\mathrm{conv}(P)$ with the property that the circumcircle of every triangle contains the whole set P.

Exercise 2 (10 Points)

In the lecture it was shown that the Lawson flip algorithm finds a Delaunay triangulation of a point set with n points after at most $O(n^2)$ flips starting from any triangulation. Show that this bound is tight, i.e., that there is a family of point sets and a triangulation of them such that the Lawson flip algorithm has to perform at least $\Omega(n^2)$ flips.

Hint: Consider a point configuration as shown in Figure 1, often called a *double chain*. First prove that one has to perform at least $\Omega(n^2)$ flips to go from the first triangulation of the polygon defined by the two chains to the other triangulation of the polygon (as shown in the figure). Then argue why this result carries over to triangulations of the point sets.

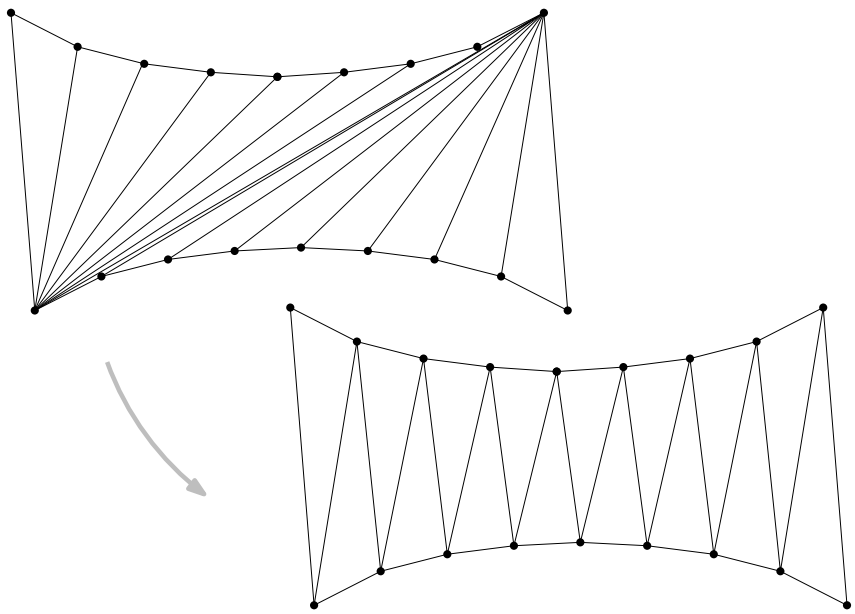

Figure 1: A triangulation and the Delaunay triangulation of a polygon formed by two concave chains.

Exercise 3 (10 Points)

Fix a circle in the plane. For any n distinct points on this circle we get a drawing of the complete graph $K_n = ([n] = \{1, \ldots, n\}, \binom{[n]}{2})$ by connecting any two of the points by a line segment. The interior of the circle is partitioned into several regions by this drawing.

Assume that the points are placed such that no three line segments intersect in a common intersection point. What is the number of regions?

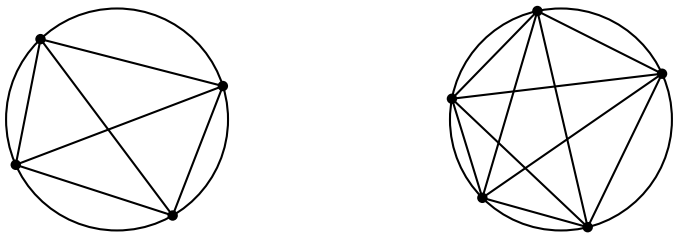

Figure 2: Drawings of K_4 and K_5 - giving 8 and 16 regions, respectively

Exercise 4 (30 Points)

Prepare a short presentation (between 4 and 6 minutes) of the survey you did in the previous homework. You will have an opportunity to give a trial talk in an exercise session. This trial talk will *not* be graded and gives you a chance to receive feedback from the assistant and your fellow students. In the following exercise session, you will then give the talk once more in front of a larger audience and your performance will be graded.

General guidelines. This exercise has two goals. Firstly, you should learn how to put together an interesting presentation. This means that you must do some research. Get an overview about the topic at hand: What is it all about, and why is it interesting? (You have already done this in the last homework.) Furthermore you have to develop an idea of what to present and how to present it, which brings us to the second goal: Your audience should learn something worthwhile by attending your presentation. You should seriously think about what you want this to be. Talks that have not been prepared with this goal in mind can be very painful to endure and this will reflect in the grading.

Preparation. *Use electronic slides!* It doesn't matter whether you use Powerpoint, Keynote, Slitex, Beamer, or anything else. These days, the only serious alternative to electronic slides are pure blackboard talks, but since your talk will be 4-6 minutes, giving a blackboard talk is probably not a good idea. Well-prepared slides guide you through your talk almost automatically. Choose suitable colors as some combinations (e.g. green colors on white background) are either invisible or hurt the eye on some projectors. It is a good idea to test your presentation on the projector before giving the graded talk to make sure the colors show up as expected. You will have a chance to do this during the trial talk.

Show many figures! It is difficult for the audience to understand even simple definitions without an illustration. You will have prepared your presentation well and (hopefully) know it by heart, but the audience hears it for the first time. Pictures add redundancy, so they allow the audience to cross-check whether their understanding of the previous formal definition was correct. Explaining pictures also slows down the presentation and gives people the time

necessary to absorb the material. Depending on your topic, you should consider having a picture on (almost) every slide. A useful drawing editor for geometric figures is Ipe.

Use a large font, and show one thing at a time! A slide only helps if it corresponds to what the speaker is telling at the very moment. Slides that stay on for minutes while the speaker is simply droning on makes the audience lose attention. There may be technical slides (that explain an algorithm, say) which are necessarily somewhat denser, but these should be exceptions. Every slide should focus on just one issue.

Practice your presentation! It's not easy to judge the length of a presentation without actually giving it. Once you have practised it a few times you'll now how much time you need to explain each part and can be sure that you stay within the allotted time. This is also a good way to become more confident and less nervous. You can practice with an audience (e.g. with a trial talk in the exercise session) but also without one.

Content. The essential rule here is: keep it simple, but do the simple things well. We expect you to stay within the allotted time of 4-6 minutes and there is only so much you can say in such a short time. So it makes sense to prefer simple over complicated material. The time will not allow you to explain the topic comprehensively. You may pick the most interesting results or one interesting direction to present. Be prepared to answer questions about your topic, in particular about any terms you mention on your slides.

Due date: 7.11.2013, 13h15