Eidgenössische Technische Hochschule Zürich
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Algorithms, Probability, and Computing **Exercises KW46** **HS17**

General rules for solving exercises

- When handing in your solutions, please write your exercise group on the front sheet:

  Group A/B: Wed 13–15 CAB G 56

  Group C: Wed 16–18 CAB G 52

- This is a theory course, which means: if an exercise does not explicitly say "you do not need to prove your answer", then a formal proof is **always** required.

The following exercises will be discussed in the exercise class on November 15, 2016. Please hand in your solutions not later than November 14.

Exercise 1: Finding a Separating Line

Let $R, B \subseteq \mathbf{R}^2$ be given finite sets ("red and blue points"). A *(strictly) separating line* is a line ℓ with the property that all red points lie strictly on one side of ℓ, and all blue points strictly on the other side.

Formulate a linear program such that, given an optimal solution of your LP, you can decide if a separating line exists and, if so, compute one.

Exercise 2: Fitting a Ball into a Convex Polytope

Let $H_1, \dots, H_m$ be halfspaces in $\mathbf{R}^n$ given by $H_i = \{\mathbf{x}\colon \mathbf{a}_i^T\mathbf{x} \leq b_i\}$ where $\mathbf{a}_i \in \mathbf{R}^n$ and $b_i \in \mathbf{R}$. We want to find the largest n-dimensional ball that is completely contained in the intersection $\bigcap_{i=1}^m H_i$, which is assumed to be non-empty.

Formulate a linear program with variables $\mathbf{c} \in \mathbf{R}^n$ and $r \in \mathbf{R}$ whose optimal solution is the center point $\mathbf{c}^*$ and radius r^* of this largest ball.

Exercise 3: Linear Programs in Equational Form

Show that every linear program can also be converted into the following *equational form*:

$$\text{maximize } \mathbf{c}^{\mathsf{T}}\mathbf{x} \text{ subject to } A\mathbf{x} = \mathbf{b},\ \mathbf{x} \geq \mathbf{0}.$$

What is the maximum increase in the number of variables and in the number of constraints in such a transformation?

Exercise 4: Maximum Number of Vertices of 3-dimensional Convex Polytopes

Let P be a convex polytope which is defined as the intersection of n given closed half-spaces in $\mathbf{R}^3$. Show that the number of vertices of P is at most $2n-4$.

HINT: Use Euler's formula for plane graphs, $v - e + f = 2$.

Exercise 5: Certificates for Infeasibility of Systems of Linear Equations

(Preparation for chapter 6.5.)

Prove that a system $A\mathbf{x} = \mathbf{b}$ of linear equations is unsolvable if and only if there is $\mathbf{y}$ with $A^{\mathsf{T}}\mathbf{y} = \mathbf{0}$ and $\mathbf{b}^{\mathsf{T}}\mathbf{y} = 1$.