ETH
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Institute of Theoretical Computer Science
Mohsen Ghaffari, Angelika Steger, David Steurer, Emo Welzl, Peter Widmayer

Algorithms, Probability, and Computing In-Class Exercises KW42 HS17

The following exercises will be discussed in the exercise class on October 18, 2017. Since we expect you to be working on the special assignment, all exercises are in-class.

In-Class Exercise 1: Locating a Point in a Line Arrangement

Recall the algorithm for counting the points below a query line: We interpreted the question in the dual setting, so we were looking for the number of lines above a query point. Prove the space bound in lemma 2.6, which was left as an exercise:

$$\sum_{v \text{ inner node}} |\bar{S}_v| \;\leq\; 2n^2$$

where, as you may recall,

- n is the number of lines,
- v ranges over the nodes of the data structure used in the algorithm, which has one node for every level of the line arrangement;
- S_v is the set of x-coordinates of the corresponding level;
- $\bar{S}_v$ is the 'enhanced' set: If a node v has no child which is an inner node, $\bar{S}_v = S_v$. Otherwise, $\bar{S}_v$ is obtained from S_v by adding every other value from each of the sets $\bar{S}_u$, u a non-leaf child of v.

In-Class Exercise 2: Nearest Neighbor Changes

We are given a set P of n points in $\mathbf{R}^2$ and a point q which has distinct distances to all points in P. We add the points of P in random order (starting with the empty set), and observe the nearest neighbor of q in the set of points inserted so far. What is the expected number of distinct nearest neighbors that appear during the process?

In-Class Exercise 3: Checking Matrix Multiplication

Suppose $K = GF(2)$ (i. e., all calculations with matrices are done modulo 2), and suppose the matrix C is wrong in exactly one row. Show that in one iteration the success probability of detecting an error in the supposed product matrix C is exactly $\frac{1}{2}$.

In-Class Exercise 4: The Schwartz-Zippel Theorem is Tight

Given a finite set S of rational numbers and positive integers d and n, $d \leq |S|$, find a polynomial $p(x_1, x_2, \dots, x_n)$ of degree d for which the Schwartz–Zippel theorem is tight. That is, the number of n-tuples $(r_1, \dots, r_n) \in S^n$ with $p(r_1, \dots, r_n) = 0$ is $d|S|^{n-1}$.

In-class Exercise 5: The Permanent and the Determinant

Let A be an $n \times n$ matrix with $0/1$-entries. For $1 \leq i, j \leq n$ let $\epsilon_{i,j}$ be independent random variables, $\epsilon_{i,j} \in_{\text{u.a.r.}} \{-1, +1\}$. Let B be the random matrix with $b_{i,j} = \epsilon_{i,j} \cdot a_{i,j}$. In other words, to get B from A we randomly assign signs to the entries of A.

(a) Show that $\mathbf{E}[\det B] = 0$.

(b) Show that $\mathbf{E}\left[(\det B)^2\right] = \text{per}(A)$.