ETH

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Algorithms, Probability, and Computing — Exercises KW49 — HS17

The following exercises will be discussed in the exercise class on December 6, 2017. Please hand in your solutions not later than December 13.

In class exercise 1: Reducing the Number of Colors in a Single Round

In Lemma 8.5, we saw a single-round algorithm for reducing the number of colors exponentially. Here, we discuss another such method, which transforms any k-coloring of any rooted-tree to a $2\log k$-coloring, so long as $k \geq C_0$ for a constant C_0.

The method works as follows. Let each node u send its color $\phi_{old}(u)$ to its children. Now, each node v computes its new color $\phi_{new}(v)$ as follows: Consider the binary representation of $\phi_{old}(v)$ and $\phi_{old}(u)$, where u is the parent of v. Notice that each of these is a $\log_2 k$-bit value. Let i_v be the smallest index i such that the binary representations of $\phi_{old}(v)$ and $\phi_{old}(u)$ differ in the i^{th} bit. Let b_v be the i_v^{th} bit of $\phi_{old}(v)$. Define $\phi_{new}(v) = (i_v, b_v)$. Prove that $\phi_{old}(v)$ is well-defined, and that it is a proper $(2\log k)$-coloring.

In class exercise 2: 7-Coloring Planar Graphs

In the lecture we briefly mentioned how to 7-color planar graphs in the LOCAL model in $O(\log n)$ rounds. In this exercise we go through the argument more thoroughly.

(a) Show that in a planar graph the number of vertices of degree at least 7 is at most $\frac{6}{7}n$.

(b) How can you orient the edges of a planar graph in $O(\log n)$ rounds so that every vertex has at most 6 outgoing edges?

(c) Show that when given such an orientation we can 3^6-color the planar graph in $O(\log^* n)$ additional rounds.

(d) How to turn this coloring into a 7-coloring of the planar graph in $O(\log n)$ rounds?

(e) Do the same arguments extend to an arbitrary graph with at most cn edges where c is a constant?