ETH
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Algorithms, Probability, and Computing — Exercises KW44 — HS18

General rules for solving exercises

- When handing in your solutions, please write your exercise group on the front sheet:

  Group A: Wed 13–15 CAB G 56

  Group C: Wed 16–18 CAB G 52

- This is a theory course, which means: if an exercise does not explicitly say "you do not need to prove your answer", then a formal proof is **always** required.

The following exercises will be discussed in the exercise class on October 31, 2018. Please hand in your solutions not later than October 30.

Exercise 1

Prove that a system $A\mathbf{x} = \mathbf{b}$ of linear equations is unsolvable if and only if there is $\mathbf{y}$ with $A^{\mathsf{T}}\mathbf{y} = \mathbf{0}$ and $\mathbf{b}^{\mathsf{T}}\mathbf{y} = 1$.

Exercise 2

(a) Explain why the $\mathbf{y}$ as in Lemma 4.5 indeed certifies the nonexistence of a nonnegative solution.

(b) Prove that all of the three variants of the Farkas lemma, I–III, are mutually equivalent.

Exercise 3

Let $\mathbf{c} \in \mathbb{R}^n$ be some fixed vector and let $k \in \{1, \ldots, n\}$ be an integer. We denote by $\mathbf{1} \in \mathbb{R}^n$ and by $\mathbf{0} \in \mathbb{R}^n$ the vectors of all-ones and all-zeros, respectively. Let further x be a vector of n real valued variables. For the questions (a)-(c) describe explicitly the set of all basic feasible solutions and find an optimal basic feasible solution to the linear program. Your answer should of course be phrased in terms of the parameters $\mathbf{c}$ and k.

(a) $\min \mathbf{c}^\mathsf{T}\mathsf{x}$ subject to $\mathbf{0} \leq \mathsf{x} \leq \mathbf{1}$.

(b) $\min \mathbf{c}^\mathsf{T}\mathsf{x}$ subject to $\mathbf{1}^\mathsf{T}\mathsf{x} = 1$ and $\mathsf{x} \geq \mathbf{0}$.

(c) $\max \mathbf{c}^\mathsf{T}\mathsf{x}$ subject to $\mathbf{1}^\mathsf{T}\mathsf{x} \leq k$ and $\mathbf{0} \leq \mathsf{x} \leq \mathbf{1}$.