Notation

$\mathbf{Z}$ denotes the set of integers, $\mathbf{N}$ the positive integers (natural numbers), and $\mathbf{N}_0$ the non-negative integers. For $i, j \in \mathbf{Z}$, $\{i..j\} := \{n \in \mathbf{Z} \mid i \leq n \leq j\}$. and $[j] := \{1..j\}$.

$\mathbf{Z},\mathbf{N},\mathbf{N}_0$ $\{i..j\}$ $[j]$

$\mathbf{R}$ denotes the set of real numbers, $\mathbf{R}^+$ the positive real numbers, and $\mathbf{R}_0^+$ the non-negative real numbers. $e = 2.71828...$ is[1] the base of the natural logarithm ln. For $b \in \mathbf{R}^+$, $\log_b$ is the logarithm base b; log is short for $\log_2$.

$\mathbf{R},\mathbf{R}^+,\mathbf{R}_0^+$ e $\ln,\log_b,\log$

A, B sets; $k \in \mathbf{N}_0$: $|A|$ denotes the cardinality of A, 2^A the power set of A (set of all subsets of A), $\binom{A}{k}$ the set of all k-element subsets of A, $A \oplus B$ the symmetric difference, and $A \times B$ the Cartesian product. The set of functions $A \longrightarrow B$ is denoted by B^A.

$|A|$, 2^A $\binom{A}{k}$, $A \oplus B$ $A \times B$ B^A

$\Pr[\mathcal{E}]$ denotes the probability of event $\mathcal{E}$. $\mathbf{E}[X]$ denotes the expected value of random variable X. "u.a.r." is short for "uniformly at random." If an element x is drawn u.a.r. from a set Ω we write $x \in_{\text{u.a.r.}} \Omega$, i.e. $\Pr[x = \omega] = \frac{1}{|\Omega|}$ for all $\omega \in \Omega$. For P a predicate, $[P]$ is 1 if P holds, and 0, otherwise (*indicator function*).

$\Pr[\mathcal{E}], \mathbf{E}[X]$ u.a.r. $\in_{\text{u.a.r.}}$ $[P]$

A (simple undirected) *graph* is a pair $G = (V, E)$, where V is a finite set (the *vertices*) and $E \subseteq \binom{V}{2}$ (the *edges*). Vertex u is termed *adjacent* to vertex v (or u is a *neighbor* of v) if $\{u, v\} \in E$.

A (simple) *directed graph* is a pair $G = (V, E)$, where V is a finite set and $E \subseteq V \times V$ (the *directed edges*[2] or *arcs*). For $u, v \in V$, u is an *out-neighbor* of v if $(v, u) \in E$, and u is an *in-neighbor* of v if $(u, v) \in E$; u is a *neighbor* of v (or *adjacent* to v) if it is an out- or in-neighbor of v.

[1] Frequently we use the inequality $1 + x \leq e^x$ for all $x \in \mathbf{R}$.

[2] In a directed graph (as defined here), a vertex can have an edge to itself (a *loop*), while this is outruled for undirected graphs.