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Computational Geometry — Exercise Set 2 — HS08

URL: http://www.ti.inf.ethz.ch/ew/courses/CG08/

Exercise 1

A set $S \subset \mathbb{R}^d$ is *star-shaped* $\iff$ there exists a point $c \in S$, such that for every point $p \in S$ the line segment $\overline{cp}$ is contained in S. A set $S \subset \mathbb{R}^d$ is a *pseudotriangle* $\iff$ it is a simple polygon and has exactly three convex vertices (see Figure 1).

In the following we consider subsets of $\mathbb{R}^d$. Prove or disprove:

a) Every star-shaped set is convex.

b) Every convex set is star-shaped.

c) The intersection of two convex sets is convex.

d) The union of two convex sets is convex.

e) The intersection of two star-shaped sets is star-shaped.

f) The intersection of a convex set with a star-shaped set is star-shaped.

g) Every pseudotriangle is star-shaped.

Exercise 2

Let $P = \{p_1, \ldots, p_n\}$ be a set of $n \geq 3$ points in $\mathbb{R}^2$ and let $q \in \text{conv}(P)$ be another point. Prove that there exist three points p_i, p_j and p_k, $1 \leq i, j, k \leq n$, such that $q \in \text{conv}(\{p_i, p_j, p_k\})$.

see reverse

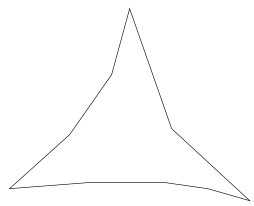

Figure 1: A pseudotriangle

Exercise 3

Consider three points $p, q, r \in \mathbb{R}^2$, given by their Cartesian coordinates $p = (p_x, p_y)$, $q = (q_x, q_y)$ and $r = (r_x, r_y)$. Show: the sign of the determinant

$$\begin{vmatrix} 1 & p_x & p_y \\ 1 & q_x & q_y \\ 1 & r_x & r_y \end{vmatrix}$$

determines if r lies to the right, to the left or on the directed line through p and q.

Exercise 4

Let $P \subset \mathbb{R}^2$ be a convex polygon, given as an array $\texttt{p[0]} \ldots \texttt{p[n]}$ of its $n+1$ vertices in counter clockwise order.

(a) Describe an algorithm with running time $O(\log(n))$, which determines whether a point q lies inside, outside or on the boundary of P.

(b) Describe an algorithm with running time $O(\log(n))$, which finds a (right) tangent to P from a query point q outside P (i.e. you should find a vertex $p[i]$, s.t. whole P is contained in a (left) halfplane determined by the line $qp[i]$).