Eidgenössische Technische Hochschule Zürich
Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
Swiss Federal Institute of Technology Zurich

Institute for Theoretical Computer Science
Dr. B. Gärtner, Dr. M. Hoffmann and Marek Sulovský

6.10.2008

Computational Geometry — Exercise Set 4 — HS08

URL: http://www.ti.inf.ethz.ch/ew/courses/CG08/

Exercise 1

Let P be a convex n-gon and μ be a measure on all the $\binom{n}{3}$ possible triangles formed by its vertices. Give an algorithm which finds a triangulation T minimizing

$$\max\{\mu(\Delta) \mid \Delta \text{ is a triangle of } T\}$$

the weight of the "heaviest" triangle of T in time $O(n^3)$.

Exercise 2

Consider the $n \times n$ grid. Look at the paths from the lower left corner $(0,0)$ to the upper right corner (n,n) passing on the grid edges and using exactly $2n$ of them (i.e. the shortest paths).

(a) How many are there all such paths?

(b) How many are there such paths, which pass above the diagonal at least once, i.e. use some point with coordinates $(i, i+1)$ (*Hint:* Find a bijection between such lower-diagonal paths on the $n \times n$ grid and all paths on the $(n-1) \times (n+1)$ grid [those you can easily count the same way as in (a)]; for this you can look at the first edge, which goes above the diagonal and after that, switch the direction of the following edges sending the upward edges to the right and right edges upward).

(c) Conclude that the number of all the paths in $n \times n$ grid, which stay below the diagonal is $\frac{1}{n+1}\binom{2n}{n}$.

Exercise 3

How many are there strings consisting of n left brackets "(" and n right brackets ")", forming a mathematically correct expression (i.e. it is not allowed to close more brackets, than were opened to that moment)?

Exercise 4

(a) Using the Euler formula derive that every planar graph on n vertices has at most $3n - 6$ edges.

(b) Prove that every planar graph has a vertex of degree at most 5.