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Computational Geometry Exercise Set 5 HS08

URL: http://www.ti.inf.ethz.ch/ew/courses/CG08/

Exercise 1

Consider the lifting map p from the plane to the unit paraboloid $\mathcal{U} = \{(x, y, z) \in \mathbb{R}^3 \mid z = x^2+y^2\}$ given by $\ell(x, y) := (x, y, x^2 + y^2)$. Let C be a circle in $\mathbb{R}^2$. Show that there is a hyperplane h_C such that

a) the lifting $\ell(C)$ of the circle C (i.e. $\{\ell(p) \mid p \in C\}$) is the set $\mathcal{U} \cap h_C$

b) the lifting of the interior of the circle C is the set $\mathcal{U} \cap h_C^-$ where h_C^- denotes the lower open halfplane of the hyperplane h_C.

Exercise 2

The Euclidean minimum spanning tree (EMST) of a finite point set $M \subset \mathbb{R}^2$ is a spanning tree for which the sum of the edge lengths is minimum (among all spanning trees of M). Show:

a) Every EMST of M contains a closest pair, i.e. an edge between two points $p, q \in M$, that have minimum distance to each other among all point pairs in $\binom{M}{2}$.

b) Every Delaunay Triangulation of M contains an EMST of M.

Exercise 3

Show that every simple polygon has a triangulation.

Recall: A polygon is an area bounded by a closed path consisting of finitely many line segments. A polygon is called simple if its sides do not intersect. A triangulation of a polygon is a triangulation whose unbounded face is the complement of the polygon.