Eidgenössische Technische Hochschule Zürich
Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
Swiss Federal Institute of Technology Zurich

Institute for Theoretical Computer Science
Dr. B. Gärtner, Dr. M. Hoffmann and Marek Sulovský

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Computational Geometry — Exercise Set 7 — HS08

URL: http://www.ti.inf.ethz.ch/ew/courses/CG08/

Exercise 1

For a sequence of n pairwise distinct numbers $y_1, \ldots, y_n$ consider the sequence of pairs $(\min\{y_1, \ldots, y_i\}, \max\{y_1, \ldots, y_i\})_{i=0,1,\ldots,n}$ ($\min \emptyset := +\infty, \max \emptyset := -\infty$). How often do these pairs change in expectation if the sequence is permuted randomly, each permutation appearing with the same probability? Determine the expected value.

Exercise 2

The non-vertical geometric duality transform is a mapping assigning to non-vertical lines points and vice versa. To a point $a \in \mathbb{R}^2$ it assigns the line

$$a^* := \{x \in \mathbb{R}^2 \mid x_2 = a_1 x_1 - a_2\}$$

and to a non-vertical line l, which can be uniquely written in a form $l = \{x \in \mathbb{R}^d \mid x_2 = a_1 x_1 - a_2\}$, it assigns a point $l^* := a \in \mathbb{R}^2$.

1. Show that this mapping preserves incidences, i.e. for a point a and a line l it holds $a \in l \iff l^* \in a^*$.
2. Show that this mapping preserves order, i.e. for a point a and a line l it holds: a is above $l \iff l^*$ is above a^*.
3. Describe the image of the following point sets under this mapping
   (a) a half plane
   (b) $k \geq 3$ colinear points
   (c) a line segment
   (d) the boundary points of the upper convex hull of a finite point set.

Exercise 3

Find an algorithm, which solves the following problem in polynomial time (find the fastest you are able to): Given a set of closed halfplanes containing the origin 0 in their interior, find their intersection.