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Computational Geometry — Exercise Set 8 — HS08

URL: http://www.ti.inf.ethz.ch/ew/courses/CG08/

Exercise 1

You are given

- a star-shaped polygon $P \subset \mathbb{R}^2$, represented as a doubly connected list of its vertices $V(P)$,
- and a point $c \in P$ (not necessarily in $V(P)$), such that for all $p \in P$ the line segment $\overline{cp}$ is contained in P.

Describe an algorithm which triangulates P in linear time. The algorithm could for example output all edges of the triangulation, that are not already edges of the polygon.

Exercise 2

Let L be a set of n lines in $\mathbb{R}^2$ no three of which pass through a common point. Suppose that all lines from $P \subseteq L$ are parallel to each other, no two lines from $L \setminus P$ are parallel to each other, and no line from $L \setminus P$ is parallel to those from P. Determine the number of vertices, edges, and faces of the arrangement $\mathcal{A}(L)$ in terms of n and $k := |P|$.

Exercise 3

For an arrangement $\mathcal{A}$ of a set of n lines in $\mathbb{R}^2$, let $\mathcal{F} := \bigcup_{C \text{ is cell of } \mathcal{A}} \overline{C}$ denote the union of the closure of all bounded cells. Show that the complexity (number of vertices and edges of the arrangement lying on the boundary) of $\mathcal{F}$ is $O(n)$.

Exercise 4

Given a set of lines in the plane with no three intersecting in a common point, form a graph G whose vertices are the intersections of the lines, with two vertices adjacent if they appear consecutively on one of the lines. Prove that $\chi(G) \leq 3$. (χ is the chromatic number of the graph)