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Computational Geometry — Exercise Set 9 — HS08

URL: http://www.ti.inf.ethz.ch/ew/courses/CG08/

Exercise 1

The 3–sum′ problem is defined as follows: given 3 sets S_1, S_2, S_3 of n integers each, are there $a_1 \in S_1, a_2 \in S_2, a_3 \in S_3$ such that $a_1 + a_2 + a_3 = 0$?

Prove that this 3–sum′ and 3–sum as defined on the lecture (there the sets were the same) are equivalent. By equivalence we mean that 3-sum can be reduced to 3-sum′ and vice-versa, 3–sum′ can be reduced to 3–sum.
By reduction we mean, that any instance I of size n of one can be transformed in subquadratic time into an instance I' of size $O(n)$ of the other, such that I has a solution if and only if I' has a solution.

Note: Such an equivalence implies that if there was a subquadratic algorithm for 3–sum then there would be a subquadratic algorithm for 3–sum′ and vice-versa.

Exercise 2

Prove that $\lambda_2(n) = 2n - 1$.

Exercise 3

In *Segment Splitting*, we are given a set of n line segments and have to decide whether there exists a line that does not intersect any of the segments but splits them into two non-empty subsets.

To show that this problem is 3-Sum-hard, we can use essentially the same reduction as for GeomBase, where we interpret the points along the three lines $y = 0$, $y = 1$, and $y = 2$ as sufficiently small “holes”. The parts of the lines that remain after punching these holes form the input segments for the Splitting problem.

Show that it is enough to punch holes of length $\varepsilon < f(\delta)$ centered at the corresponding point of the GeomBase reduction, where δ denotes the distance between a closest pair of points. The function f should be as large as you can achieve.