Institute for Theoretical Computer Science
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Computational Geometry — Homework 2 — HS08

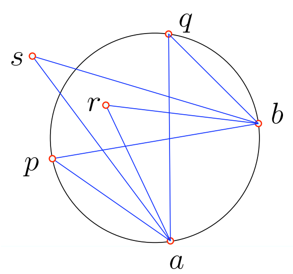
URL: `http://www.ti.inf.ethz.ch/ew/courses/CG08/`

Exercise 1

For a triangle Δ, denote by $\alpha_1(\Delta), \alpha_2(\Delta), \alpha_3(\Delta)$ its three angles (in arbitrary order). Let T be a triangulation of a finite point set P in the plane and let $\sigma(T)$ be the sorted (increasing) sequence of all its angles $\alpha_i(\Delta)$ for all triangles $\Delta \in T$ and $i \in \{1, 2, 3\}$.

Show that for P in general position (no four points cocircular), the Delaunay triangulation lexicographically maximizes the sequence σ among all the triangulations (consequently, it also maximizes the minimal angle).

Hint: Take an arbitrary triangulation and analyze how the Lawson flip algorithm changes the angle sequence (it should be lexicographically nondecreasing). You might want to use a fact from the high school geometry depicted on the figure below:



$$\angle arb > \angle apb = \angle aqb > \angle asb$$

Exercise 2

Let $P \subseteq \mathbb{R}^2$ be a finite point set and $G = (P, E)$ a plane graph with vertex set P (we thus consider the edges $e \in E$ as line segments). A triangulation $\mathcal{T}$ of P is said to *respect* G if it contains all segments $e \in E$.

A triangulation $\mathcal{T}$ of P that respects G is said to be a *constrained Delaunay triangulation* of P with respect to G if the following holds for every triangle Δ of $\mathcal{T}$:

> The circumcircle of Δ contains only points $q \in P$ in its interior that are not *visible* from the interior of Δ. A point $q \in P$ is visible from the interior of Δ if there exists a point p in the interior of Δ such that the line segment $\overline{pq}$ does not intersect any segment $e \in E$. We can thus imagine the line segments $e \in E$ as "blocking the view".

This latter property is referred to as the *constrained empty circle property*. If $E = \emptyset$, this coincides with the notion of the "normal" Delaunay triangulation.

Constrained Delaunay triangulations are useful if you would like to have a Delaunay triangulation, but certain edges are already prescribed. For example, if you want a Delaunay triangulation of a simple polygon. You may not be able to get a proper Delaunay triangulation with all triangles satisfying the empty circle property, but in a sense, a constrained Delaunay triangulation is as close as you can get to a proper Delaunay triangulation if you are forced to include the edges in E.

Here is the actual problem: Prove that for every point set P and every plane graph $G = (P, E)$, there exists a constrained Delaunay triangulation of P with respect to G. Moreover, describe a polynomial algorithm that computes such a triangulation.

Due date: 30.10.2008, 13h00