Eidgenössische Technische Hochschule Zürich

Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
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Institute for Theoretical Computer Science *6.11.2008*
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Computational Geometry Homework 3 HS08

URL: http://www.ti.inf.ethz.ch/ew/courses/CG08/

Exercise 1

Objective of this exercise will be developing a data structure for so-called range counting for halfspaces.

a) Given a set P of n points in the plane in general position, show that it is possible to partition this set by two lines such that each region contains at most $\lceil \frac{n}{4} \rceil$ points.

b) Find a data structure of size $O(n)$, which can be constructed in time $O(n \log n)$ and allows you, for any halfspace h, to output the number of points $|P \cap h|$ of P contained in this halfspace h in time $O(n^{\alpha})$ for some $0 < \alpha < 1$. For this you will need claim a).

Exercise 2

The goal of this exercise will be applying the configuration spaces for yet another problem — of sorting n real numbers.

You are given a set X of n distinct real numbers and your goal is sorting them into ascending order.

a) Define a configuration space over X such that it is possible to construct the sorted sequence from the active configurations with respect to X (you might need to have a data structure connecting the active configurations just like the doubly connected edge lists for the convex hulls in $\mathbb{R}^3$).

b) Describe a randomized incremental algorithm, which constructs the set $T(X)$ of active configurations (together with the data structures you need) and analyze the expected runtime in the configuration space framework. The runtime should be $O(n \log n)$.

Exercise 3

Given a simple polygon P with n vertices, describe a data structure (with space and time needed to construct it polynomial in n) to report for any query point q inside P the "scenery visible from q", that is, the cyclic sequence $S(q)$ of polygon edges visible from q. Query time should be $O(\log n + |S(q)|)$.

For the best grade, the time needed to construct the data structure should not be more than $O(n^4 \log^k n)$ for some small power k.

Due date: 20.11.2008, 13h00