Computational Geometry

Definition, Application Areas, and Course Overview

Computational Geometry

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- started out by developing solid theoretical foundations, but became more and more applied over the last years

Application Areas

- Computer Graphics
- Computer-aided design / manufacturing
- Telecommunication
- Geology
- Architecture
- Geographic Information Systems
- VLSI design (chip layout)
- ...

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  - important algorithms and data structures
  - design patterns

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- does not require you to program anything
- requires you to solve homeworks in a careful mathematically correct way
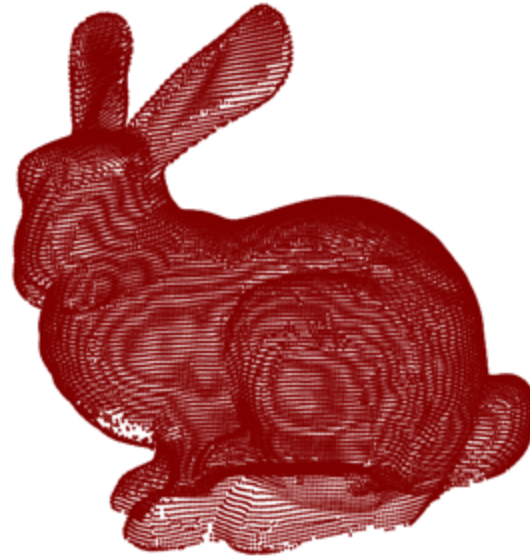
Surface Reconstruction

- Digitizing 3-dimensional objects
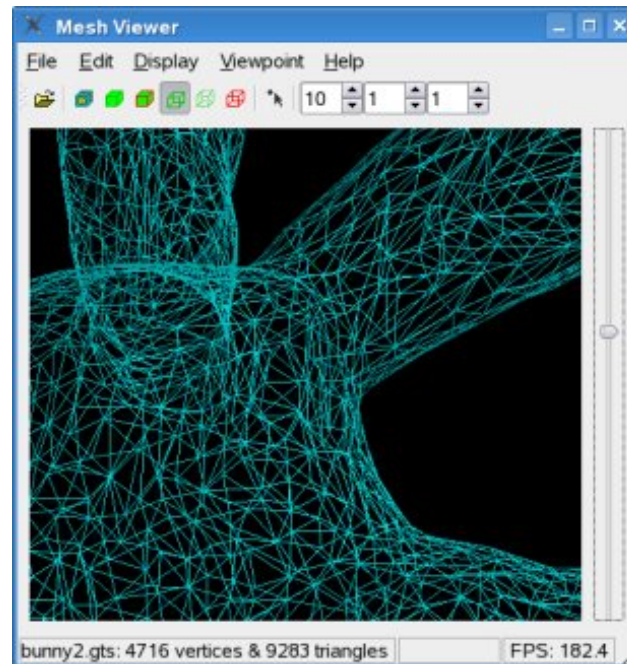

Stanford Bunny

Surface Reconstruction

- Step 1: Scan the object (3d laser scanner)

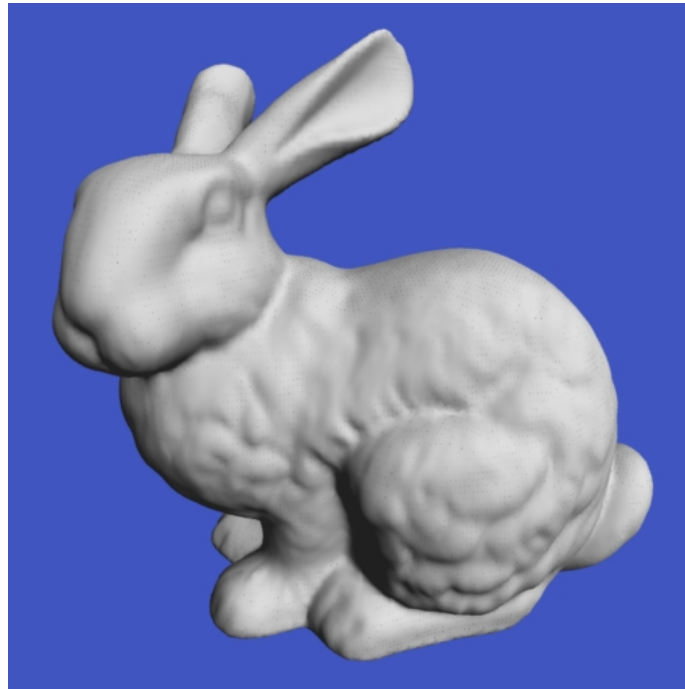*set of points in $\mathbf{R}^3$*

Surface Reconstruction

- Step 2: Create a triangulation

set of triangles in $\mathbf{R}^3$

Surface Reconstruction

- Step 3: process the triangulation (rendering)

smooth surface in $\mathbf{R}^3$
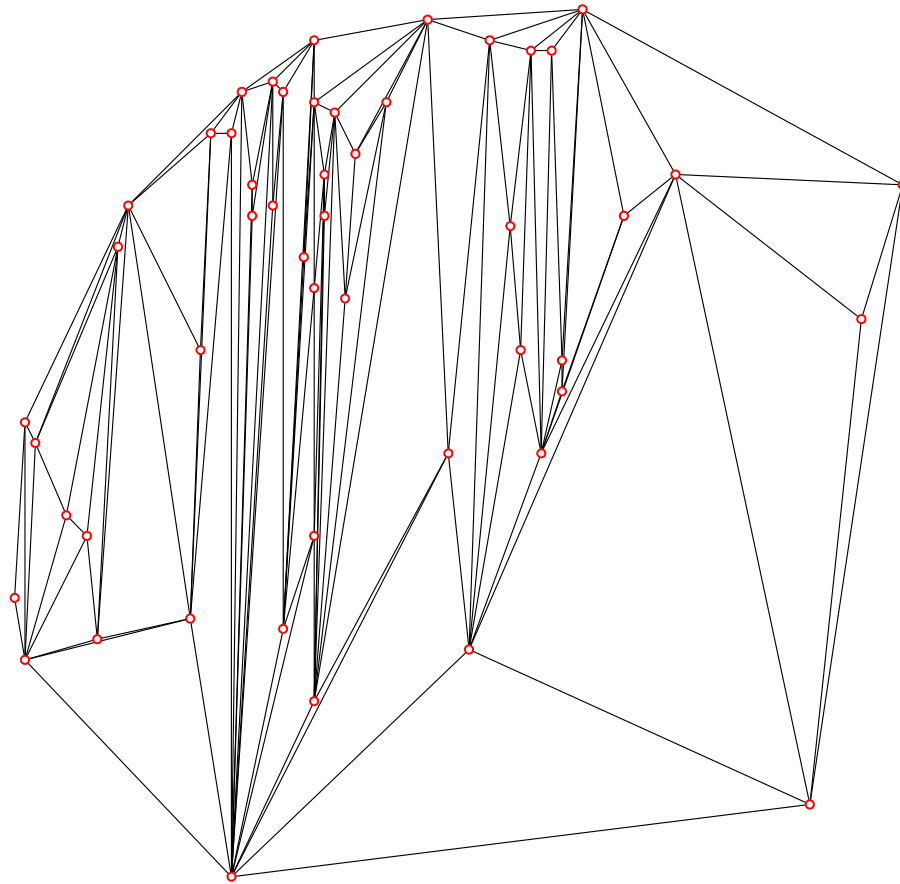
Surface Reconstruction

- Major Computational Geometry task:
    - Create a "good" triangulation

In this Course:
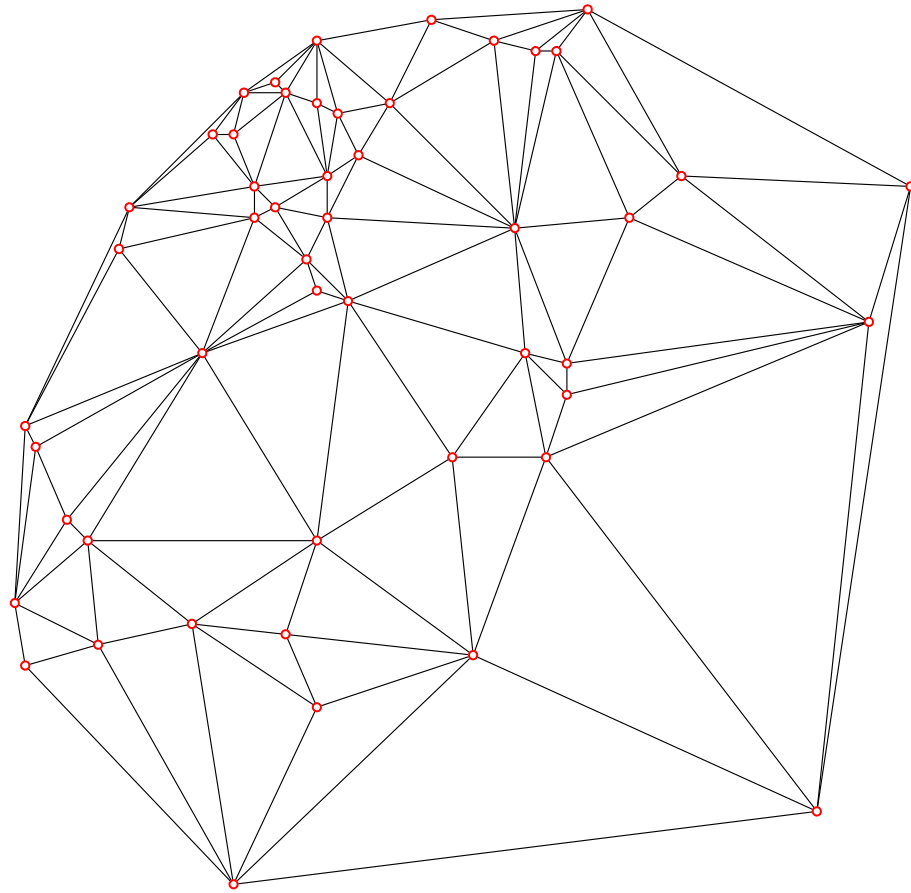# Good and bad triangulations in R^2

bad triangulation
(long and skinny
triangles)

In this Course:
Good and bad triangulations in R^2

good triangulation
(no small angles,
almost regular
triangles)

Collision detection

Check whether two (possibly complicated) 3d objects intersect!

Collision detection

- Bounding volume heuristic:
    - Approximate the objects by simple ones that enclose them (bounding volumes)

Collision detection

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  - popular bounding volumes: boxes, spheres, ellipsoids,...

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  - *only* if bounding volumes intersect, apply more expensive intersection test(s)

In this Course:
Smallest enclosing ball

- Given: finite point set in R^d

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Smallest enclosing ball

- Given: finite point set in R^d
- Wanted: the smallest ball that contains all the points

Miniball
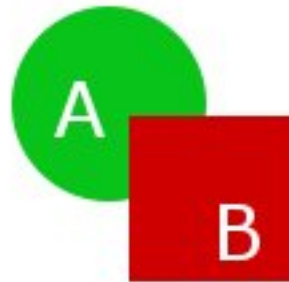*V2.0*

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Smallest enclosing ball

- Given: finite point set in R^d
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popular free software (also some commercial licenses sold):

`http://www.inf.ethz.ch/personal/gaertner/miniball.html`

Miniball V2.0

Boolean Operations

- Given two (2d,3d) shapes, compute their...
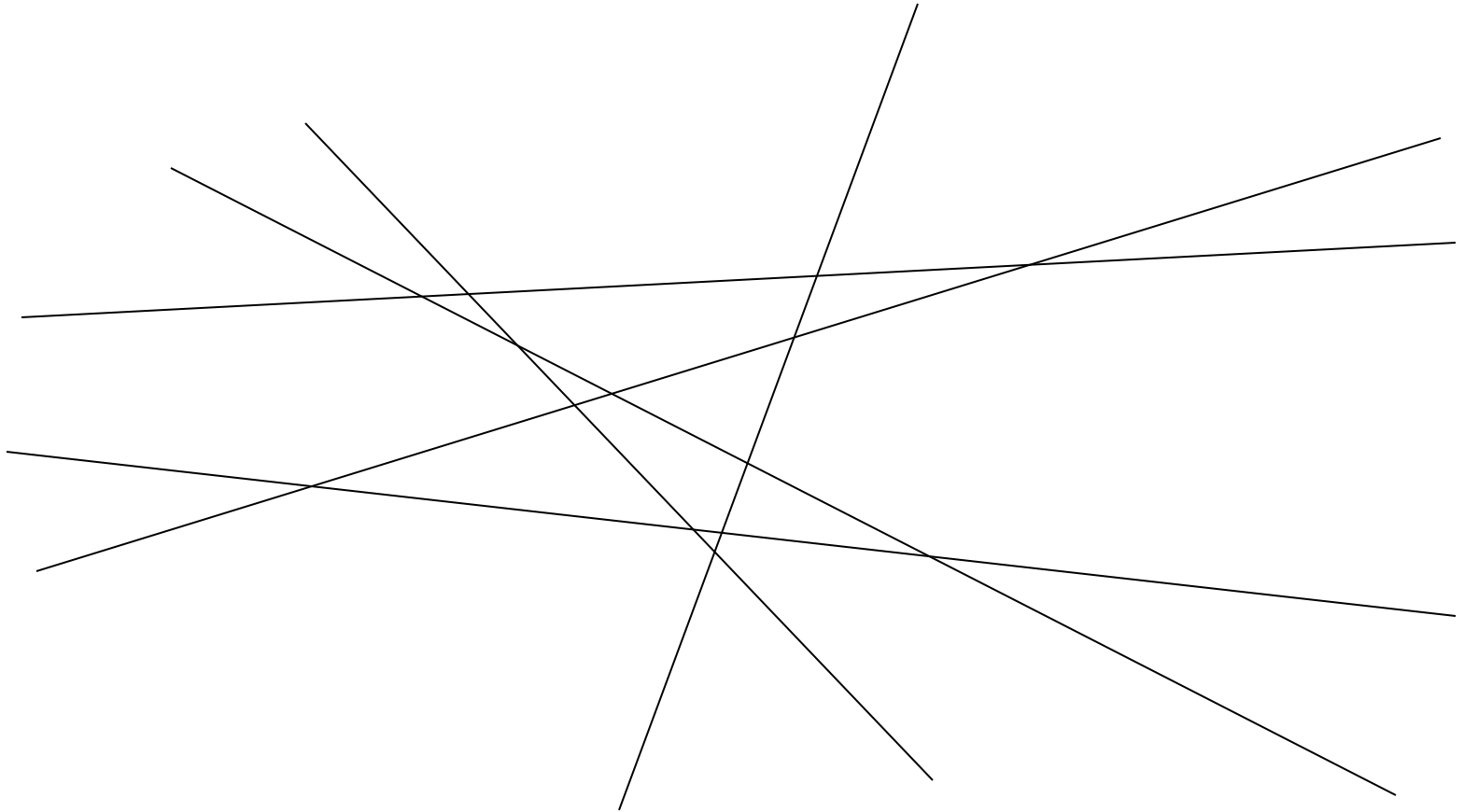

Intersection   Subtraction   Union

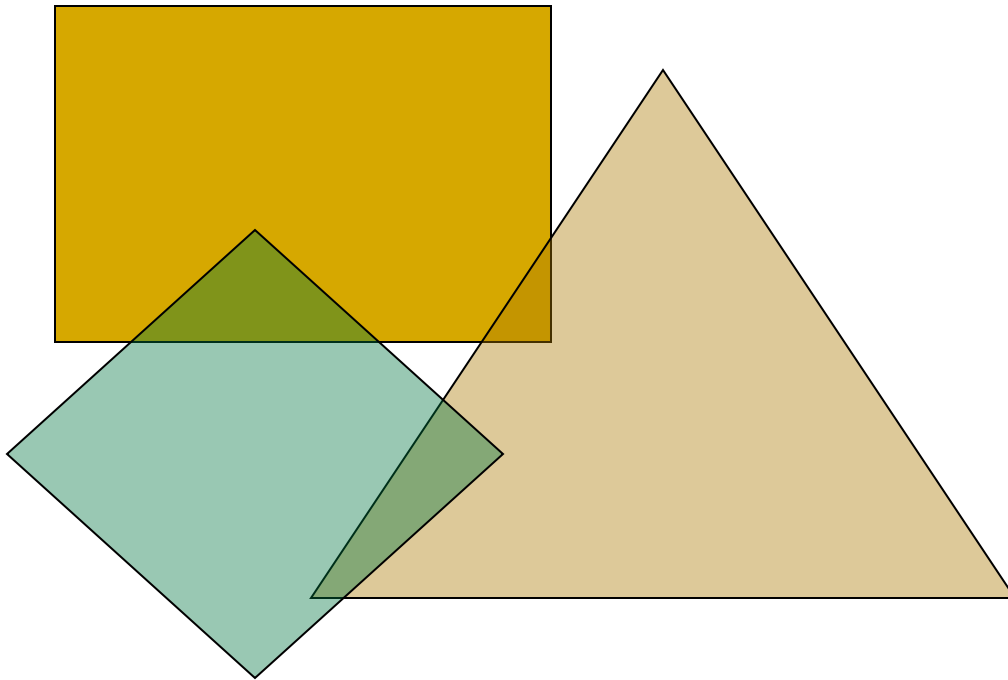# Boolean Operations

- ubiquituous in *computer-aided design*
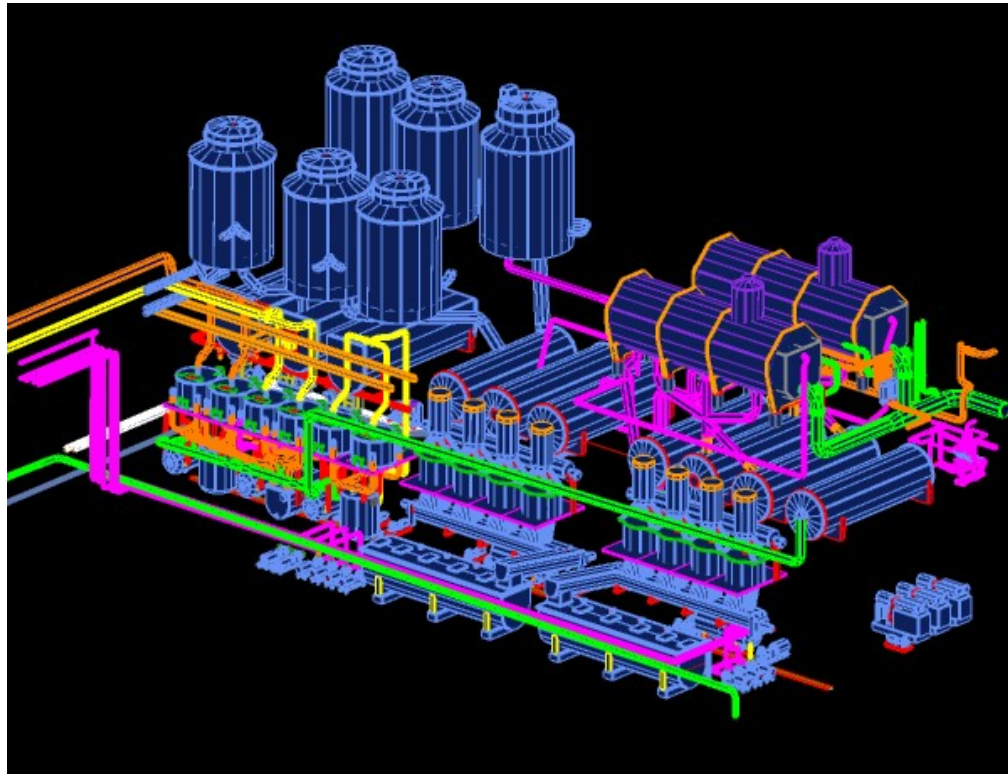
In this Course: Arrangements of lines
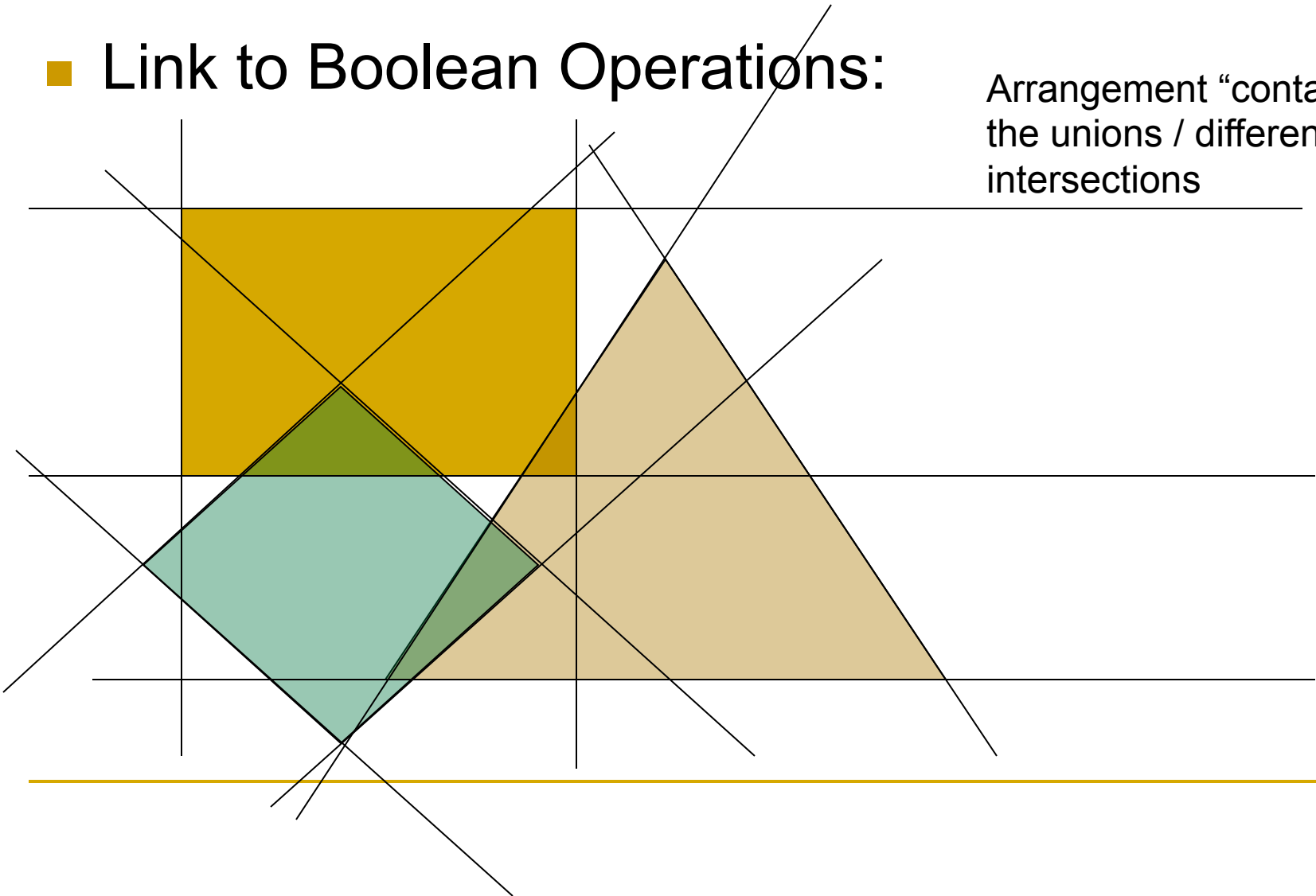
In this Course: Arrangements of lines

- Link to Boolean Operations:

In this Course: Arrangements of lines

- Link to Boolean Operations:

Arrangement "contains" all the unions / differences / intersections

In this Course:
Topic Overview

- **Convex Hulls**
- **Line Sweep (design pattern)**
  - for segment intersections
- **Triangulations**
  - of point sets
  - Delaunay Triangulations
- **Point Location**
  - Trapezoidal Maps
  - Voronoi Diagrams
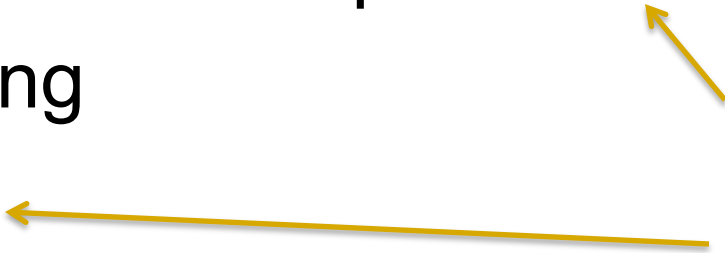
In this Course: Topic Overview

- Pseudotriangulations
- Linear Programming
- Smallest Enclosing Balls
- Line Arrangements
- Davenport-Schinzel sequences
- Motion planning
- Epsilon Nets

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# Topic Overview

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- Linear Programming
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discrete and combinatorial geometry

A Teaser Puzzle

- Suppose that we independently choose three points uniformly at random from a circle. What is the probability that the triangle defined by them contains the center of the circle?
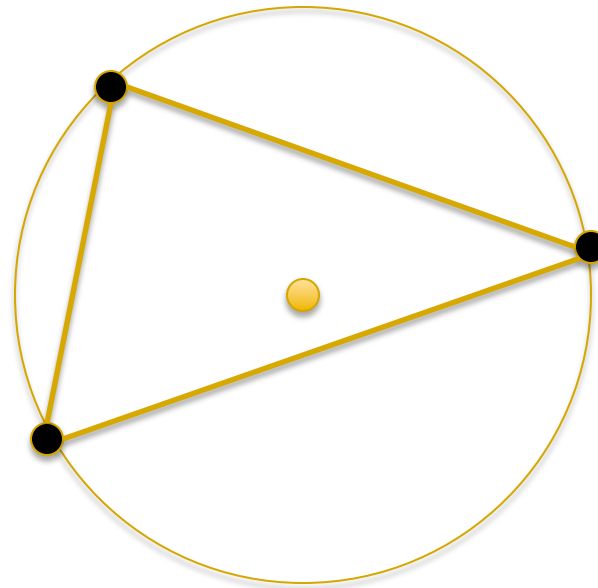
Formalities

- Weekly Exercise session + exercise sheets (marked, but not graded)

- 3 graded homeworks, to be submitted in LaTeX (usually two weeks time)   30%

- Homeworks may include an oral presentation or a small research project

- 30 minute oral exam, with 30 minutes preparation time; one question will be from the weekly exercise sheets;   70%