*Institute for Theoretical Computer Science* *20.09.2012*
*Dr. B. Gärtner, Anna Gundert, and Dr. M. Hoffmann*

# Computational Geometry — Exercise Set 1 — HS12

URL: http://www.ti.inf.ethz.ch/ew/courses/CG12/

---

**Exercises:** Every week you are supposed to look at the exercises you find in the lecture notes. You are advised to solve them and hand them in to the assistant for corrections and suggestions.

There will be three special series of exercises, called homework, which will be obligatory and graded. These three grades will contribute 10% each to the final grade.

---

**Remark:** Throughout the course we will be using asymptotic notation when analyzing algorithms. In this exercise we want to make sure you are familiar with it.

Let $g : \mathbb{N} \to \mathbb{R}$. We denote

$$O(g(n)) = \{f : \mathbb{N} \to \mathbb{R} \mid \exists\ c > 0 \text{ and } n_0 \in \mathbb{N} \text{ such that } 0 \leq f(n) \leq c \cdot g(n) \text{ for all } n \geq n_0\}.$$

and similarly

$$\Omega(g(n)) = \{f : \mathbb{N} \to \mathbb{R} \mid \exists\ c > 0 \text{ and } n_0 \in \mathbb{N} \text{ such that } 0 \leq c \cdot g(n) \leq f(n)0 \text{ for all } n \geq n_0\}.$$

Finally

$$\Theta(g(n)) := O(g(n)) \cap \Omega(g(n))$$

Denote a base 2 logarithm by log. We define the iterated logarithm $\log^{(i)}$ (for $i \in \mathbb{N}$)

$$\log^{(i)} n := \begin{cases} n & , i = 0 \\ \log \log^{(i-1)} n & , \text{otherwise} \end{cases}$$

and

$$\log^* n := \min\{i \geq 0 \mid \log^{(i)} n \leq 1\}$$

which essentially determines how many times a logarithm needs to be applied until we reach 1.

## Exercise 1

Order the following functions by their order of growth, i.e., into a sequence $g_1, \dots, g_{15}$ s.t. $g_i \in O(g_j)$ for $i \leq j$.

| | | | | |
|---|---|---|---|---|
| $2^{\log^* n}$ | $n^2$ | $n!$ | $(\log n)!$ | $\log^* \log n$ |
| $n^{72}$ | $\log \log^* n$ | $4^{\log n}$ | $1$ | $(\log n)^{\log n}$ |
| $(\frac{n}{3})^n$ | $n^{\log \log n}$ | $\log n$ | $e^n$ | $n \log n$ |

## Exercise 2

Determine the order of magnitude of

$$\sum_{i=1}^{n} \frac{1}{i}.$$

## Exercise 3

Let $A = \{a_1, a_2, \ldots, a_n\}$ be a set of real numbers. Assume that two numbers $a_i$ and $a_j$ can be compared (i.e., $a_i \leq a_j$ can be decided) in constant time.

a) Give a randomized algorithm that finds the median of $A$ in expected run time $O(n)$.

b) Prove that the probability that your algorithm takes $\lambda$ times longer than the expected run time is at most $1/\lambda$.

## Exercise 4

Find an algorithm to decide whether a point lies inside or outside a simple polygon. More precisely, given a simple polygon $P$ as a list of its vertices $(v_1, v_2, \ldots, v_n)$ in counter clock wise order and a query point $q$, decide whether $q$ is inside $P$ (which includes the possibility that $q$ is on the boundary of $P$) or completely outside. The runtime of your algorithm should be of order $O(n)$.