Eidgenössische Technische Hochschule Zürich
Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
Swiss Federal Institute of Technology Zurich

Institute for Theoretical Computer Science
Dr. B. Gärtner, A. Gundert and Dr. M. Hoffmann

22.11.2012

Computational Geometry — Homework 3 — HS12

URL: http://www.ti.inf.ethz.ch/ew/courses/CG12/

For the following two exercises we define a simple polygon P *to be* star-shaped *if there exists a point* s **in the interior** *of* P *such that for any point* p *on the boundary of* P *the ray from* s *through* p *intersects the boundary of* P **only at** p. *The point* s *is called a* star-point.

Exercise 1 (12 points)

Show that for an n-vertex polygon, given as a list of its vertices in counterclockwise order, we can decide in $O(n)$ time whether it is star-shaped and compute a star-point (as defined above) if it exists.

Exercise 2 (12 points)

Prove that a star-shaped polygon, given as a sequence of $n \geq 3$ vertices in counterclockwise order and a star-point, can be triangulated in $O(n)$ time.

For the next two exercises suppose we are given two sets of points in $\mathbb{R}^2$*, one set representing cancer cells, the other one representing healthy cells. With radiation therapy we want to destroy cancer cells without damaging healthy cells. You can assume that the set of all (cancer and healthy) cells is in general position.*

Exercise 3 (12 points)

First we want to find out whether we can destroy all cancer cells without touching healthy cells. Show how to decide whether there is a disk containing all cancer cells in the interior such that the healthy cells lie strictly outside and compute the center and radius of such a disk. Your algorithm should take $O(n)$ time, where n is the total number of (healthy and cancer) cells.

Exercise 4 (12 points)

Now we want to destroy the maximum number of cancer cells without damaging any healthy cells. Show how to compute the radius and the center of a disk containing as many cancer cells as possible in the interior while avoiding the healthy cells (strictly outside!) in time $O(n_1 n_2)$, where n_1 is the number of cancer cells and n_2 the number of healthy cells.

Exercise 5 (12 points)

Prove that the problem of finding a largest disk inside a convex polygon can be formulated as a linear program.

Due date: 06.12.2012