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*November 11, 2004*

## **External Memory Algorithms and Data Structures      Problem 4**

Course Webpage: <http://www.ti.inf.ethz.ch/ew/courses/EMADS04/>

### **Topic: Buffer Trees**

Due Date: November 18, 2004 at the lecture

#### **Exercise 4.1**

Show that in an external priority queue implemented by a buffer tree the current minimum can be maintained internally using only the root buffer and the set of  $O(M)$  elements in the leftmost leaves. Conclude that find-minimum queries can be answered on-line without using any IOs.

#### **Exercise 4.2**

How do you perform a decrease-key operation in an external priority queue implemented by a buffer tree? Give an upper bound for the number of IO operations.

#### **Exercise 4.3**

By describing how to deal with duplicate removal, complete the proof of the IO bound  $O(\text{sort}(N) + R/B)$  for the pairwise rectangle intersection problem.

#### **Exercise 4.4**

In some applications one is interested only in the number of points that lie in a range rather than in reporting all of them. Such queries are often referred to as *range counting queries*. In this case we would like to avoid having an additive term of  $O(R)$  in the query time.

- a) By adapting a 1-dimensional range tree, describe how a range counting query can be performed in  $O(\log n)$  internally.
- b) Externalize this algorithm. Give an upper bound for the number of IOs.