External Memory Algorithms and Data Structures

Winter 2004/2005

Riko Jacob
Peter Widmayer

Assignments: Yoshio Okamoto

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based on a lecture in Aarhus, DK, by
Gerth Stølting Brodal and
Rolf Fagerberg

Course

Lectures:

- Based on articles.
- Theoretical.
- New stuff: 1995-2004.
- Aim: General principles and methods.

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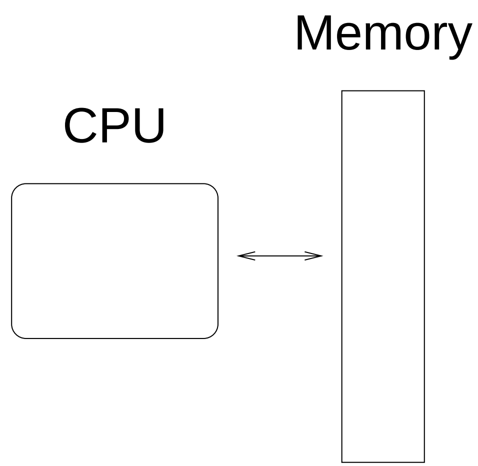
Homepage:

- `www.ti.inf.ethz.ch/ew/courses/EMADS04/`

Analysis of algorithms

The standard model:



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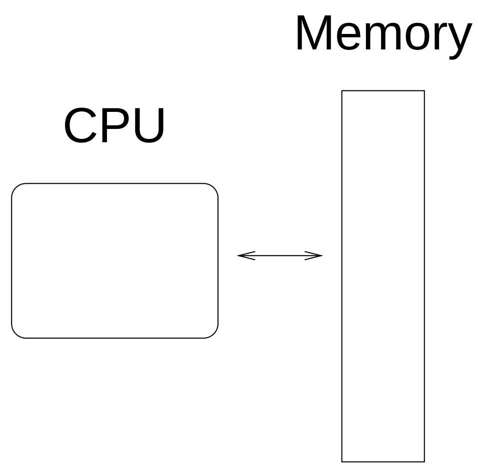
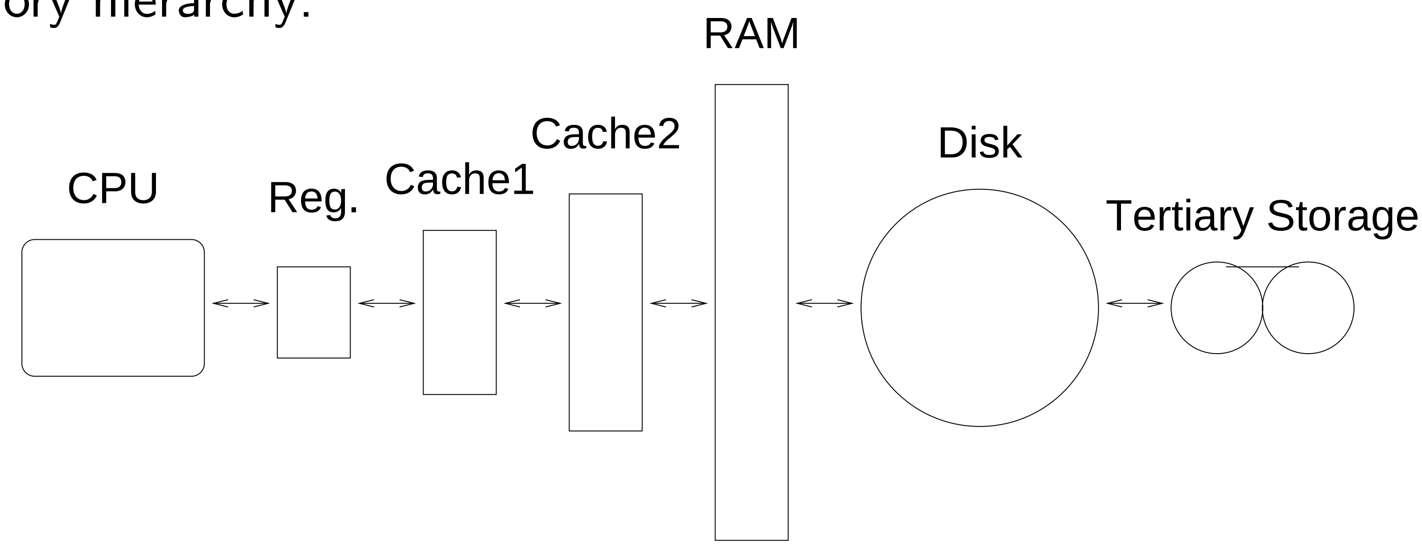


- ADD: 1 unit of time
- MULT: 1 unit of time
- BRANCH: 1 unit of time
- MEMACCESS: 1 unit of time

Reality

Memory hierarchy:



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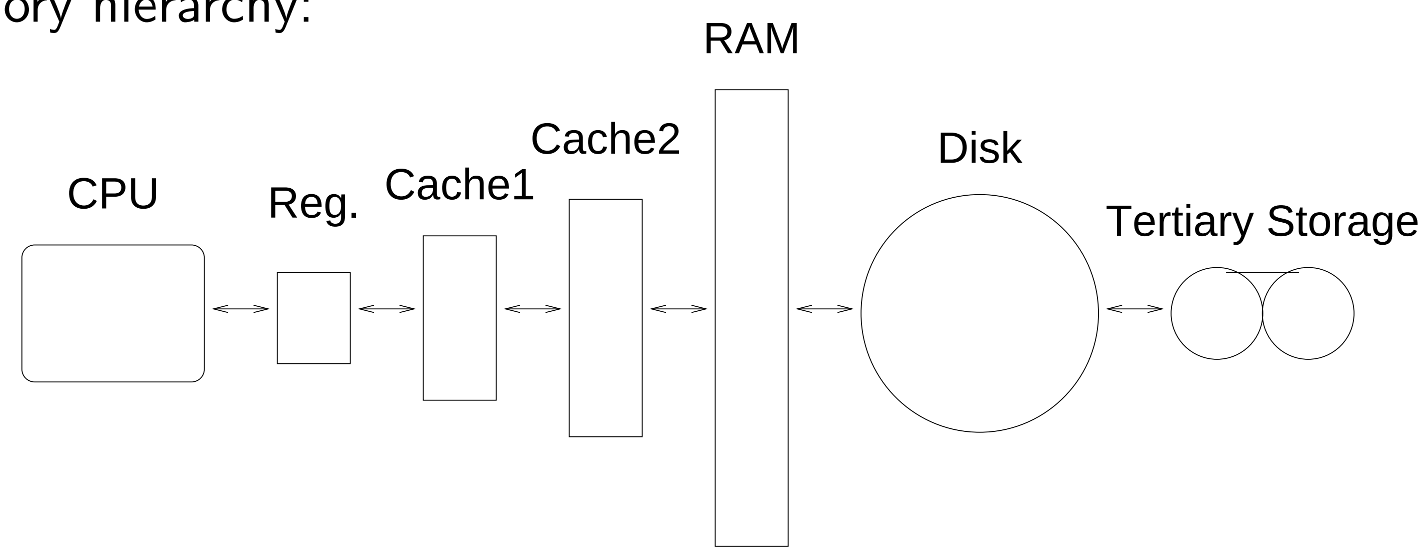

	Access time	*Volume*
Registers	1 cycle	1 Kb
Cache	5 cycles	512 Kb
RAM	50 cycles	512 Mb
Disk	20,000,000 cycles	80 Gb

Reality

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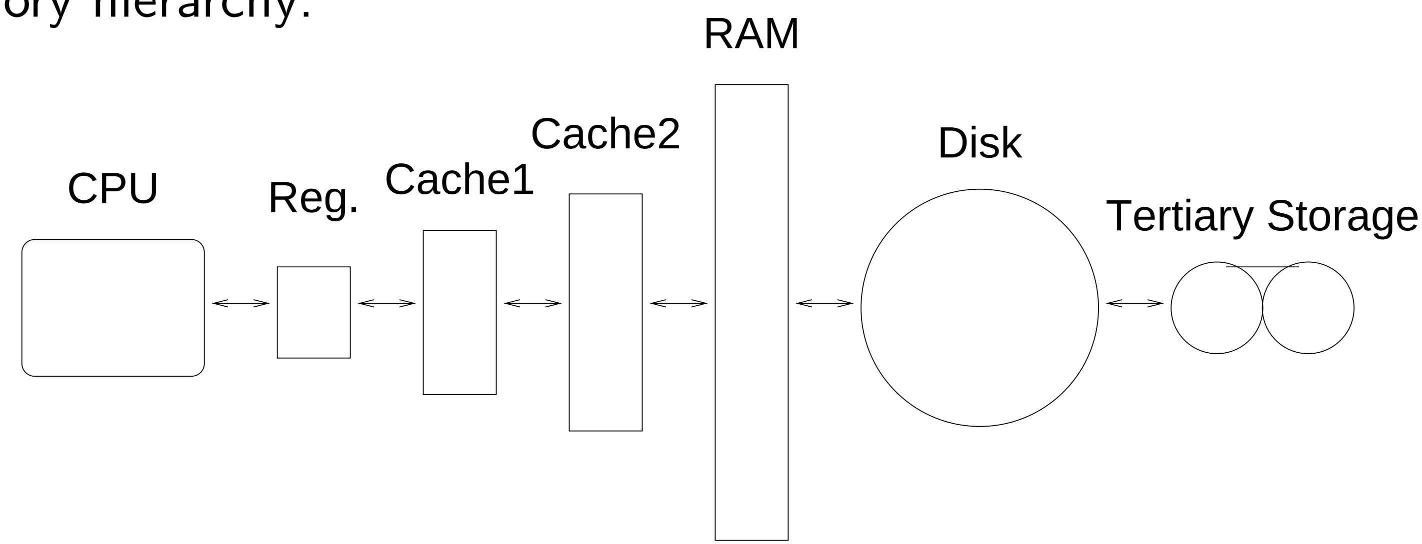

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CPU speed improves faster than RAM access time and much faster than disk access time

Reality

Many real-life problems of **Gigabyte**, **Terabyte**, and even **Petabyte** size:

- Databases
  - weather
  - geology/geograpy
  - astrology
  - financial
  - WWW
  - phone companies
- Geographic Information Systems (maps).
- Computer graphics, animation.
- VLSI design.

I/O bottleneck

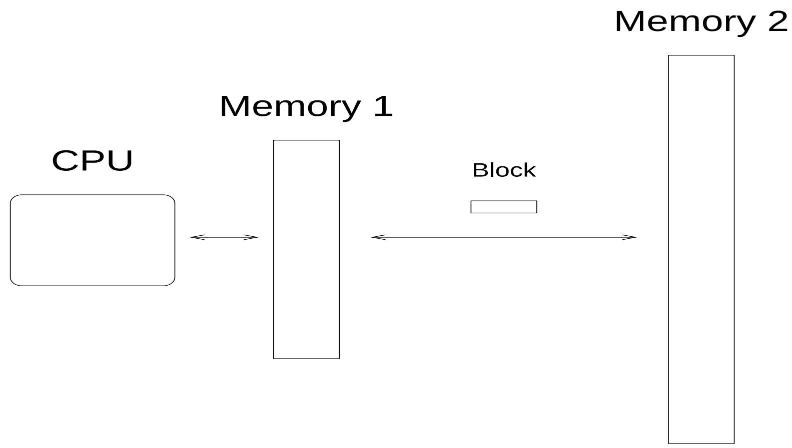
I/O is the bottleneck

$\Downarrow$

I/O should be optimized (not instruction count)

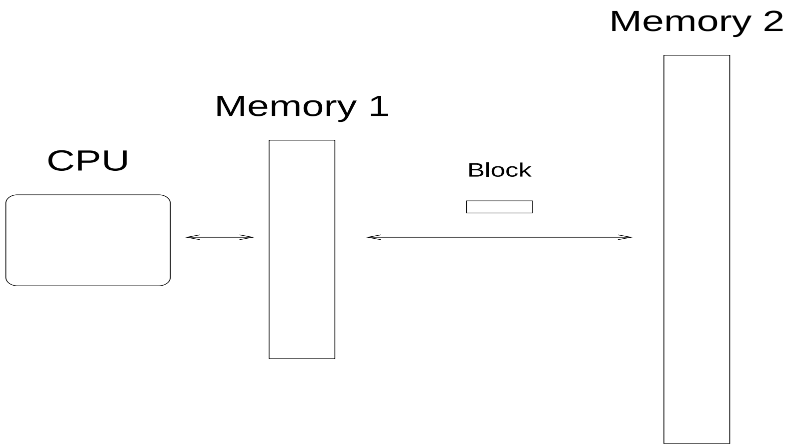
Analysis of algorithms

New **I/O-model:**



Analysis of algorithms

New **I/O-model:**



Parameters:

N	$=$	no. of elements in problem.
M	$=$	no. of elements that fit in RAM.
B	$=$	no. of elements in a block on disk.
D	$=$	no. of disks (copies of Memory 2)

Analysis of algorithms

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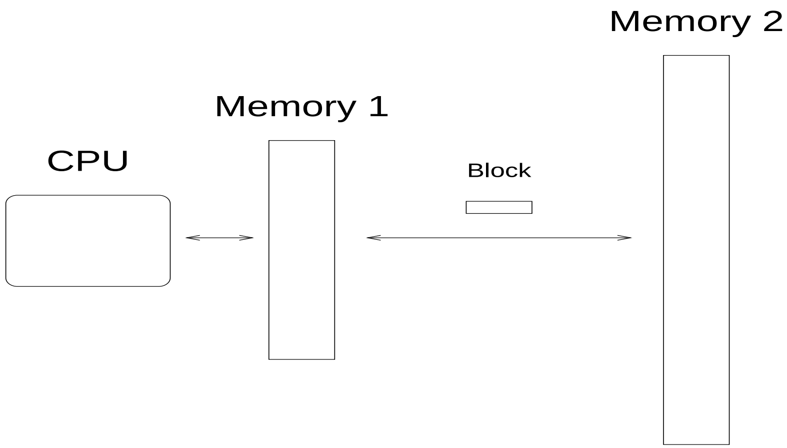

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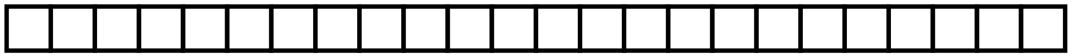
$$\begin{aligned} N &= \text{no. of elements in problem.} \\ M &= \text{no. of elements that fit in RAM.} \\ B &= \text{no. of elements in a block on disk.} \\ D &= \text{no. of disks (copies of Memory 2)} \end{aligned}$$

Cost: Number of I/O's (block transfers) between Memory 1 and Memory 2.

Generic Example

Consider two $O(n)$ algorithms:

1. Memory accessed randomly $\Rightarrow$ page fault at each memory access.
2. Memory accessed sequentially $\Rightarrow$ page fault every B memory accesses.



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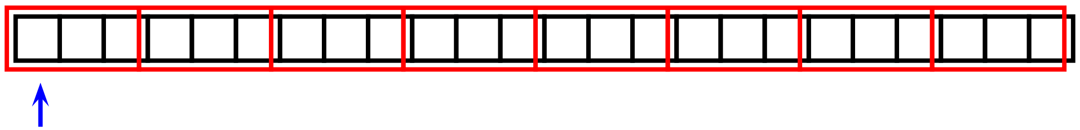

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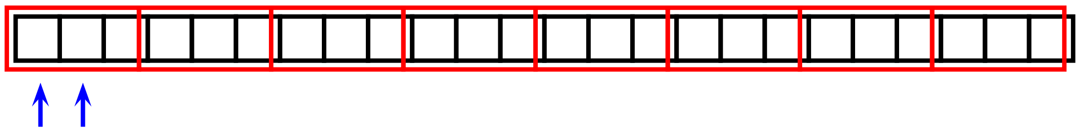

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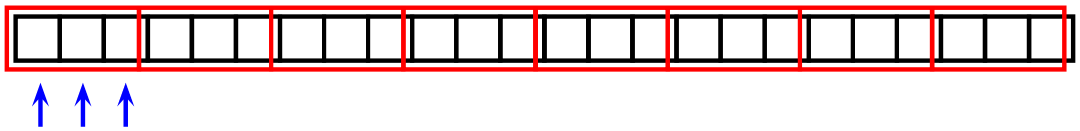

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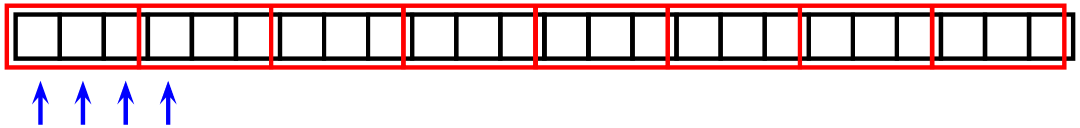

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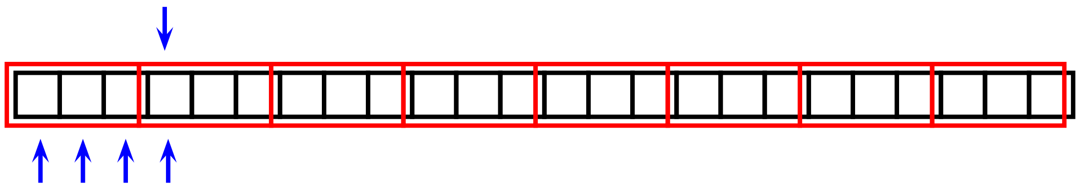

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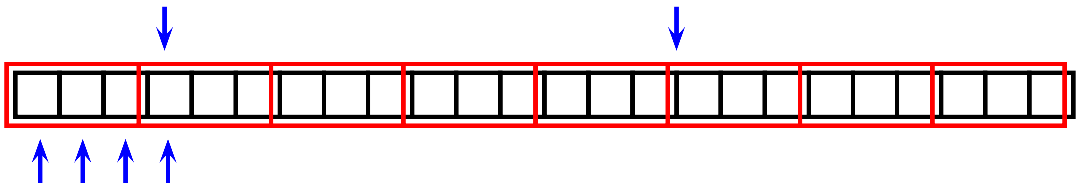

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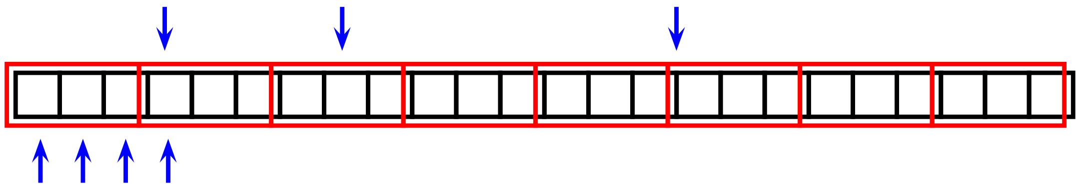

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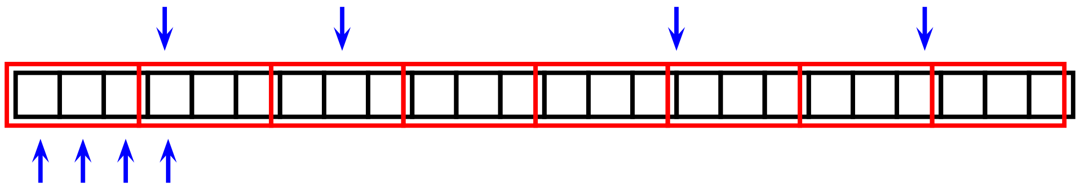

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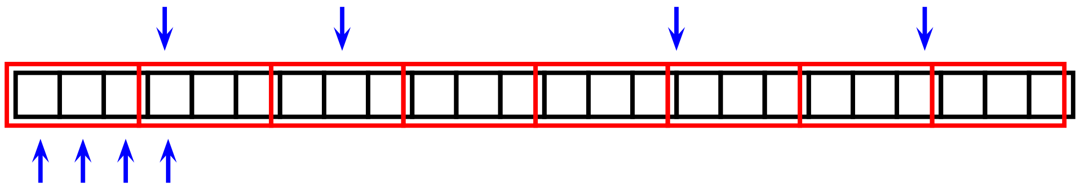

$O(N)$ I/Os *vs.* $O(N/B)$ I/Os

Typically, $B \sim 10^3$.

Specific Examples

QuickSort $\sim$ sequential access

vs.

HeapSort $\sim$ random access

QuickSort:	$O(N \log_2(N/M)/B)$
HeapSort:	$O(N \log_2(N/M))$

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Contents (approximate):

- The I/O model(s).
- Algorithms, data structures, and lower bounds for basic problems:
  - Permuting
  - Sorting
  - Searching
- I/O efficient algorithms and data structures for problems from
  - computational geometry,
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  - graphs.

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Along the way I: Generic principles for designing I/O-efficient algorithms.

Along the way II: Hands-on experience via projects.