Eidgenössische Technische Hochschule Zürich
Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
Swiss Federal Institute of Technology Zurich

*Institut für Theoretische Informatik*
*Dr. Tibor Szabó and Yoshio Okamoto*
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# *Graph Theory* — *Problem Set 3*

**Course Webpage:** `http://www.ti.inf.ethz.ch/ew/courses/GT03/`

**Due Date:** November 12, 2003 at the lecture

## Exercise 3.1
(Exercise 1.3.3 in the Textbook)

(−) Let $u$ and $v$ be adjacent vertices in a simple graph $G$. Prove that $uv$ belongs to at least $d(u) + d(v) - n(G)$ triangles in $G$. Here, a **triangle** means $K_3$.

## Exercise 3.2
(Exercise 1.3.12 in the Textbook)

(!) Prove that an even graph has no cut-edge. For each $k \geq 1$, construct a $2k+1$-regular simple graph having a cut-edge.

## Exercise 3.3
(Exercise 1.3.17 in the Textbook)

(!) Let $G$ be a graph with at least two vertices. Prove or disprove:

a) Deleting a vertex of degree $\Delta(G)$ cannot increase the average degree.
b) Deleting a vertex of degree $\delta(G)$ cannot reduce the average degree.

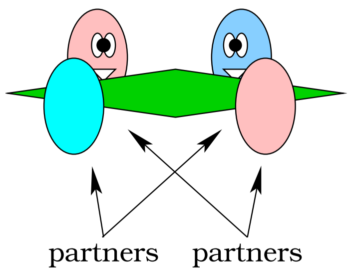
## Exercise 3.4
(Exercise 1.3.32 in the Textbook)

(!) Prove that the number of simple even graphs with vertex set $[n] = \{1, 2, \ldots, n\}$ is $2^{\binom{n-1}{2}}$. (Hint: Establish a bijection to the set of all simple graphs with vertex set $[n-1]$.)

## Exercise 3.5
(Exercise 1.3.53 in the Textbook)

(!) Each game of *bridge* involves two teams of two partners each. There is a club for *bridge*, where the following rule is in place.

> **Rule.** Once two players played as partners, they cannot play in the same game any more (neither as partner nor as opponent).




Suppose that 15 members arrive, but one decides to study graph theory. The other 14 people play until *each* has played four times. Now, the rule above starts making it difficult to schedule more games, but the members still manage to play six more games altogether.

Prove that, if the graph theorist now comes to play, then at least one more game can be scheduled. (Adapted from Bondy–Murty [1976, p111].)

## Exercise 3.6
(Exercise 1.3.50 in the Textbook)

(+) For $n \geq 3$, determine the minimum number of edges in a connected $n$-vertex graph in which every edge belongs to a triangle. (Erdős [1988])