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December 3, 2003

Graph Theory — Problem Set 7

Course Webpage: `http://www.ti.inf.ethz.ch/ew/courses/GT03/`

Due Date: December 10, 2003 at the lecture

Exercise 7.1

(Exercise 3.3.6 in the Textbook)

(!) Prove that a tree T has a perfect matching if and only if $o(T - v) = 1$ for every $v \in V(T)$. (Chungphaisan)

Exercise 7.2

(Exercise 3.3.19 in the Textbook)

(!) Let G be a 3-regular simple graph with no cut-edge. Prove that G decomposes into copies of P_4.

Exercise 7.3

(Exercise 4.1.9 in the Textbook)

For each choice of integers k, ℓ, m with $0 < k \leq \ell \leq m$, construct a simple graph G with $\kappa(G) = k$, $\kappa'(G) = \ell$, and $\delta(G) = m$. Remember to justify your construction. (Chartrand–Harary [1968])

Exercise 7.4

(Exercise 3.3.16 in the Textbook)

(!) Prove that every $(k - 1)$-edge-connected k-regular graph of even order has a 1-factor.

Exercise 7.5

(Exercise 4.1.14 in the Textbook)

(!) Let G be a connected graph in which for every edge e there are cycles C_1 and C_2 containing e whose only common edge is e. Prove that G is 3-edge-connected. Use this to show that the Petersen graph is 3-edge-connected.

Exercise 7.6

(Exercise 4.1.23 in the Textbook)

(!) Let G be an r-connected graph of even order having no $K_{1,r+1}$ as an induced subgraph. Prove that G has a 1-factor. (Sumner [1974b])