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Graph Theory — *Problem Set 12*

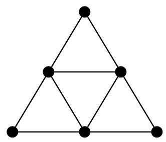
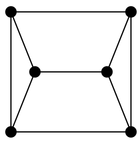
Course Webpage: `http://www.ti.inf.ethz.ch/ew/courses/GT03/`

Due Date: January 28, 2004 at the lecture

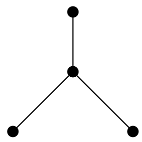
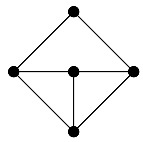
Exercise 12.1

(Exercise (a) 7.1.1, (b) not in the Textbook)

a) For each graph G below, compute $\chi'(G)$ and draw $L(G)$.

b) Show that each graph below is not a line graph of any simple graph.

Exercise 12.2

(Exercise 7.1.11 in the Textbook)

(!) Let G be a simple graph.

a) Prove that the number of edges in $L(G)$ is $\sum_{v \in V(G)} \binom{d(v)}{2}$. (Note that $\binom{a}{b} = 0$ when $a < b$.)

b) Prove that G is isomorphic to $L(G)$ if and only if G is 2-regular.

Exercise 12.3

(Exercise 7.1.15 in the Textbook)

(!) Use Tutte's 1-factor theorem to prove that every connected line graph with even number of vertices has a perfect matching. Conclude from this that the edges of a simple connected graph with even number of edges can be partitioned into paths of length 2. (Chartrand–Polimeni–Stewart [1973])

Exercise 12.4

(Exercise 7.1.26 in the Textbook)

(!) Let $k \geq 3$, and G be a k-regular graph with a cut-vertex. Prove that $\chi'(G) > k$.

Exercise 12.5

(Exercise 7.1.34 in the Textbook)

Use Petersen's Theorem to prove that $\chi'(G) \leq 3\lceil \Delta(G)/2 \rceil$ for every loopless multigraph G.

Exercise 12.6

(Exercise 7.1.33 in the Textbook)

Use Vizing's Theorem to prove that every simple graph with maximum degree Δ has an "equitable" $(\Delta+1)$-edge-coloring, i.e., a proper edge-coloring with each color used $\lceil e(G)/(\Delta+1) \rceil$ or $\lfloor e(G)/(\Delta+1) \rfloor$ times. (de Werra [1971], McDiarmid [1972])