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Graph Theory — Problem Set 13

Course Webpage: `http://www.ti.inf.ethz.ch/ew/courses/GT03/`

Due Date: February 4, 2004 at the lecture

Exercise 13.1

(Exercise not in the Textbook)

(−) Prove that every simple triangle-free planar graph G with $n(G) \geq 3$ has at most $2n(G) - 4$ edges.

Exercise 13.2

(Exercise not in the Textbook)

A (multi)graph is **outerplanar** if it has a planar embedding with every vertex on the boundary of the unbounded face. An **outerplane** (multi)graph is such an embedding of an outerplanar (multi)graph. Prove that neither K_4 nor $K_{2,3}$ is outerplanar.

Exercise 13.3

(Exercise 6.1.21 in the Textbook)

(!) Prove that a set of edges in a connected plane multigraph G forms a spanning tree of G if and only if the duals of the remaining edges form a spanning tree of G^*.

Exercise 13.4

(Exercise 6.1.25 in the Textbook)

(!) Prove that every n-vertex plane multigraph isomorphic to its dual has $2n - 2$ edges. For all $n \geq 4$, construct a simple n-vertex plane graph isomorphic to its dual.

Exercise 13.5

(Exercise 6.1.30 in the Textbook)

(!) Let G be an n-vertex simple planar graph with girth k, where k is finite. Prove that G has at most $(n-2)\frac{k}{k-2}$ edges. Use this to prove that the Petersen graph is nonplanar.

Exercise 13.6

(Exercise 6.1.35 in the Textbook)

(!) Prove that every simple planar graph with at least four vertices has at least four vertices with degree less than 6. For each even value of n with $n \geq 8$, construct an n-vertex simple planar graph G that has exactly four vertices with degree less than 6. (Grünbaum–Motzkin [1963])