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Graph Theory — *Problem Set 11*

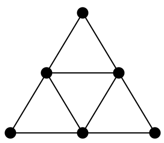
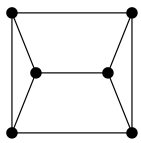
Course Webpage: `http://www.ti.inf.ethz.ch/ew/courses/GT04/`

Due Date: January 19, 2005 at the lecture

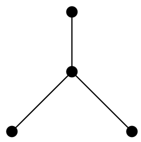
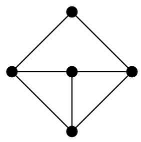
Exercise 11.1

(Exercise (a) 7.1.1, (b) not in the Textbook)

a) For each graph G below, compute $\chi'(G)$ and draw $L(G)$.

b) Show that each graph below is not a line graph of any simple graph.

Exercise 11.2

(Exercise 7.1.11 in the Textbook)

(!) Let G be a simple graph.

a) Prove that the number of edges in $L(G)$ is $\sum_{v\in V(G)}\binom{d(v)}{2}$. (Note that $\binom{a}{b}=0$ when $a<b$.)

b) Prove that G is isomorphic to $L(G)$ if and only if G is 2-regular.

Exercise 11.3

(Exercise 7.1.15 in the Textbook)

(!) Use Tutte's 1-factor theorem to prove that every connected line graph with even number of vertices has a perfect matching. Conclude from this that the edges of a simple connected graph with even number of edges can be partitioned into paths of length 2. (Chartrand–Polimeni–Stewart [1973])

Exercise 11.4

(Exercise 7.1.26 in the Textbook)

(!) Let $k\geq 3$, and G be a k-regular graph with a cut-vertex. Prove that $\chi'(G)>k$.

Exercise 11.5

(Exercise 7.1.34 in the Textbook)

Use Petersen's Theorem to prove that $\chi'(G)\leq 3\lceil\Delta(G)/2\rceil$ for every loopless multigraph G.

Exercise 11.6

(Exercise 7.1.33 in the Textbook)

Use Vizing's Theorem to prove that every simple graph with maximum degree Δ has an "equitable" $(\Delta{+}1)$-edge-coloring, i.e., a proper edge-coloring with each color used $\lceil e(G)/(\Delta{+}1)\rceil$ or $\lfloor e(G)/(\Delta{+}1)\rfloor$ times. (de Werra [1971], McDiarmid [1972])