

Assignment 0

Submission Deadline: **26 September, 2023** at 23:59

Course Website: <https://ti.inf.ethz.ch/ew/courses/LA23>

Exercises

You can get feedback from your TA for Exercise 1 by handing in your solution as pdf via Moodle before the deadline.

1. Linear combinations of vectors (hand-in) (★☆☆)

- a) Prove that every vector in \mathbb{R}^2 can be written as a linear combination of $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.
- b) Consider the two vectors $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ in \mathbb{R}^3 . Give a vector in \mathbb{R}^3 that cannot be written as a linear combination of \mathbf{v} and \mathbf{w} . Justify your answer.

2. The perfect long drink (★☆☆)

- a) Suppose that you would like to mix the perfect long drink from the two ingredients G and T . Your sources tell you that the perfect long drink is defined as 23ml of G and 77ml of T . Unfortunately, your friends already mixed two imperfect drinks: One with 15ml of G and 85ml of T , and another one with 35ml of G and 65ml of T . How can you use the two imperfect drinks to make one perfect drink?
- b) One could model the set of all possible 100ml drinks mixed from G and T as

$$D := \left\{ \begin{bmatrix} g \\ t \end{bmatrix} \in \mathbb{R}^2 : g + t = 100, g \geq 0, t \geq 0 \right\}.$$

The two imperfect drinks are then represented by the vectors $\mathbf{v} = \begin{bmatrix} 15 \\ 85 \end{bmatrix} \in D$ and $\mathbf{w} = \begin{bmatrix} 35 \\ 65 \end{bmatrix} \in D$, respectively. Using this formulation, write down the set $\hat{D} \subseteq D$ of all 100ml drinks that you could mix from \mathbf{v} and \mathbf{w} . What geometric shape does this set have?

- c) Finally, we consider the set \overline{D} of drinks of any size that can be mixed from the two drinks \mathbf{v} and \mathbf{w} . What geometric shape does \overline{D} have?