

Linear Algebra — Fall 2023
Welcome to Part II

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I am a Professor of Mathematics at ETH D-MATH, you can find me at HG G 23.1

I have been at: University of Coimbra → Princeton University → Massachusetts Institute of Technology → Courant Institute for Mathematical Sciences (New York University) → ETH Zürich



Some of my Research Interests:

High Dimensional Probability, Mathematical Statistics, Theoretical Computer Science, Combinatorics, and Optimization.

I usually teach “Mathematics of Data Science” and “Mathematics of Signals, Networks, and Learning”.

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- My [Google Scholar page](#)
- [Research Opportunities, Available Positions, and information about Thesis/Projects/Semester Papers](#)
- [Videos](#) (recorded seminars and lectures)
- [DACO schedule](#) (sometimes streamed live)

My research interests include High Dimensional Probability, Mathematical Statistics, Theoretical Computer Science, Combinatorics, and Optimization.

Calendar with events of interest (some streamed live):

Birthdays

Tuesday, 7 November	
11:30	Afonso Office Hours (HG G23.1)
12:15	Aurelio Sulser: Ramsey numbers for multiple cx
14:15	MDZ Group Meeting HG G19.1 or G19.2
Wednesday, 8 November	
17:15	Erich Baur - Random walks with reinforced mer
Tuesday, 14 November	
11:15	Afonso Office Hours (HG G23.1)
12:15	Christos Papadimitriou: Sink equilibria as a solk

Some announcements:

- My coauthors on our work on Estimation under group actions was awarded the 2023 ACHA Charles Chui Young Researcher Best Paper Award awarded by "Applied and Computational Harmonic Analysis".
- Our work on Spontaneous Synchronisation in the Kuramoto Model was featured in [Quanta Magazine](#).
- Our group has new progress on the ellipsoid problem: [arXiv:2307.01181\[math.PR\]](#), [arXiv:2310.01169\[cond-mat.dis-nn\]](#), and [arXiv:2310.05787\[math.PR\]](#).
- Our paper on Matrix Concentration and Free Probability has appeared at Inventiones Mathematicae.

Linear Algebra

Lecture Notes

Part II

(401-0131-00L at ETH Zurich)
Fall 2023

Afonso S. Bandeira
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Last update on November 6, 2023

★ Please read “READ ME” FOR PART II, the Appendix,
and MISCELLANEOUS THOUGHTS in the notes.

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— Week 7 - Part II - 2023.11.08 —

4. ORTHOGONALITY, PROJECTIONS, AND LEAST SQUARES

Guiding Question 1. If we have a system of linear equations that has no solution, how do we find the “solution” that has the smallest error? This question is central in countless applications³.

Before diving into systems of equations, we will study Projections of vectors in a subspace.

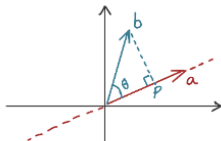
4.2. Projections.

Definition 4.2.1 (Projection of a vector onto a subspace). *The projection of a vector b on a subspace S is the point in S that is closest to b . In other words*

$$(1) \quad \text{proj}_S(b) = \underset{p \in S}{\text{argmin}} \|b - p\|.$$

Sanity Check 2. This is only a proper definition if the minimum exists and is unique. Can you show it exists and is unique? (perhaps at the end of the lecture?)

Let us build us some intuition by starting with start with projections to a line. Let S be the subspace corresponding to the line that goes through the vector a , i.e. $S = \text{Span}(a)$.



CS Lenses; and 22.12.2023 (last Friday of the semester)

- ▶ We will continue having CS Lenses exploring topics and connections outside of the immediate scope of the course; they are not relevant for the exam.
- ▶ The plan is that 22.12.2023 will be an entire lecture of CS Lenses; we plan to offer some kind of “Technical Ask Me Anything” session.

Thank You



www.afonsobandeira.com

For PhD & Postdoc positions, or research opportunities for ETH students, visit:

<https://people.math.ethz.ch/~abandeira/positions.html>

Draft available of a new book **Mathematics of Data Science**, several lecture notes, and notes with Open Problems.