D-INFK Linear Algebra HS 2024
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Assignment 0

Submission Deadline: **24 September, 2024** at 23:59
Course Website: https://ti.inf.ethz.ch/ew/courses/LA24/index.html

Exercises

You can get feedback from your TA for Exercise 1 by handing in your solution as pdf via Moodle before the deadline.

1. Linear combinations of vectors (hand-in) (★☆☆)

a) Prove that every vector in $\mathbb{R}^2$ can be written as a linear combination of $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

b) Consider the two vectors $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ in $\mathbb{R}^3$. Find a vector in $\mathbb{R}^3$ that cannot be written as a linear combination of $\mathbf{v}$ and $\mathbf{w}$. Justify your answer.

2. The perfect long drink (in-class) (★☆☆)

a) Suppose that you would like to mix the perfect long drink from the two ingredients G and T. Your sources tell you that the perfect long drink is defined as 23ml of G and 77ml of T. Unfortunately, your friends already mixed two imperfect drinks: One with 15ml of G and 85ml of T, and another one with 35ml of G and 65ml of T. How can you use the two imperfect drinks to make one perfect drink?

b) One could model the set of all possible 100ml drinks mixed from G and T as

$$D := \{\begin{bmatrix} g \\ t \end{bmatrix} \in \mathbb{R}^2 : g + t = 100, g \geq 0, t \geq 0\}.$$

The two imperfect drinks are then represented by the vectors $\mathbf{v} = \begin{bmatrix} 15 \\ 85 \end{bmatrix} \in D$ and $\mathbf{w} = \begin{bmatrix} 35 \\ 65 \end{bmatrix} \in D$, respectively. Using this formulation, write down the set $\hat{D} \subseteq D$ of all 100ml drinks that you could mix from $\mathbf{v}$ and $\mathbf{w}$. What geometric shape does this set have?

c) Finally, we consider the set $\overline{D}$ of drinks of any size that can be mixed from the two drinks $\mathbf{v}$ and $\mathbf{w}$. What geometric shape does $\overline{D}$ have?

3. Geometry of linear combinations (in-class) (★☆☆)

In this exercise, you are asked to sketch sets of points in $\mathbb{R}^3$. No formal justification is required.

a) Draw the set of linear combinations $\{\lambda_1 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + \lambda_2 \begin{bmatrix} -2 \\ 2 \\ 2 \end{bmatrix} + \lambda_3 \begin{bmatrix} -3 \\ 3 \\ 3 \end{bmatrix} \;:\; \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}\}$.

b) Draw the set of linear combinations $\{\lambda_1 \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} + \lambda_2 \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix} + \lambda_3 \begin{bmatrix} -3 \\ 3 \\ 3 \end{bmatrix} \;:\; \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}\}$.

c) Draw the set of linear combinations $\{\lambda_1 \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix} + \lambda_2 \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix} + \lambda_3 \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \;:\; \lambda_1, \lambda_2, \lambda_3 \in \mathbb{R}\}$.