Problems for the "Satisfiability of Boolean Formulas" exam
February 26, 2004, 10am–12am, HG D 3.1

Each of the six problems is granted the same number of points. Take your time, don't expect to solve all of them, even for the best grade you are not required to do so. Don't forget to put your full name on the first page, and your initials on each of the other pages.

Problem 1.1— Show that every CNF formula with one 1-clause, three 3-clauses, and one 4-clause is satisfiable.

1.2— Give an unsatisfiable CNF formula with one 1-clause, three 3-clauses, and two 4-clauses.

1.3— *Not-All-Equal-SAT.* $k \in \mathbf{N}$. Show that every k-CNF formula with less than 2^{k-1} clauses has an assignment which, for every clause, maps at least one literal to 1 and at least one literal to 0.

1.4— Show that every 9-CNF formula with 51 clauses allows an assignment which maps in each clause at least two literals to 1.

***

Problem 2.1— $k, n \in \mathbf{N}$, $2k \leq n$. How many $(2k)$-clauses over n-variables are there, where the number of positive literals equals the number of negative literals?

2.2— $n \in \mathbf{N}$, $n \geq 3$. Determine the maximum number of clauses of a 3-CNF formula over n variables which allows an assignment that satisfies at least 2 literals in each clause.

***

Problem 3— $m \in \mathbf{N}$. We are given a CNF formula with $2m$ 2-clauses all of which contain two positive literals (are of type $\{x, y\}$) and with m 3-clauses all of which contain three negative literals (are of type $\{\overline{x}, \overline{y}, \overline{z}\}$).

Show that there is an assignment that satisfies all but at most $\frac{14}{27}m$ of the $3m$ clauses.

Problem 4— Show that a satisfying assignment of a satisfiable 3-CNF formula with m constraints can be found in expected time $O(\sqrt{2}^m \text{poly}(m))$.

***

Problem 5— Suppose there is a procedure that, given a (≤ 3)-CNF formula over n variables, $r \in \mathbf{N}$, and an assignment α, searches for a satisfying assignment at Hamming distance at most r from α in time $O(2^r \cdot \text{poly}(n))$; so this is '$2^r$' here compared to '$3^r$', that was shown by procedure sb() in the course.

What would be the resulting complexity for a deterministic covering code based algorithm for finding a satisfying assignment in a (≤ 3)-CNF formula over n variables? Mention explicitly what covering radius you choose for the employed covering code.

***

Problem 6— Recall that a $(\leq \ell, 2)$-CSP is a constraint satisfaction problem where every variable has a list of at most ℓ possible colors, and constraints have size 2.

$\ell \in \mathbf{N}$, constant. Show that for every $(\leq \ell, 2)$-CSP P with m *constraints* we can construct in polynomial time a $(\leq 3, \leq \ell)$-CSP P' over m *variables* such that P is satisfiable iff P' is satisfiable. Note: It's m *constraints* versus m *variables*!