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Introduction to Topological Data Analysis **Scribe Notes 1** **HS22**

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A first introduction into topology

Definition 1. *A topological space* (X, T) *is a set of points* X*, with a system* T *of subsets of* X *(called the topology on* X*), such that*

1. $\emptyset \in T$, $X \in T$.
2. *For every* $S \subseteq T$, $\bigcup S \in T$.
3. *For every* **finite** $S \subseteq T$, $\bigcap S \in T$.

The sets in T *are called the* open *sets of* X.

Example: $X = \mathbb{R}^2$, and T the collection of open subsets (in the geometric/calculus sense) of $\mathbb{R}^2$.

Condition 3 requires the "finite": otherwise, a set $\{p\}$ consisting a single point $p \in \mathbb{R}^2$ would have to be considered to be open: it is the intersection of the infinite series of open balls of radius $1/n$ centered at p, for $n \in \mathbb{N}$.

Further examples: $(X, 2^X)$, where 2^X denotes the family of all subsets of X. This is called a *discrete topology.*

Definition 2. *A set* $Q \subseteq X$ *is called* closed*, if its complement* $X \setminus Q$ *is open. The* closure $\operatorname{cl} Q$ *is the smallest closed set containing* Q*. The* interior $\operatorname{int} Q$ *is the union of all open subsets of* Q*. The* boundary $\operatorname{bnd} Q$ *is the set minus its interior:* $\operatorname{bnd} Q = Q \setminus \operatorname{int} Q$.

Note that sets can be open and closed simultaneously: in every topological space (X, T), $\emptyset$ and X are such examples. In a discrete topology, every subset $S \subseteq X$ is open and closed.