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Theoretische Informatik (Kernfach) *SS 2005*
Exercise Set 1

Weekly exercise sets will be available online on Thursdays. Solutions can be handed in one week later during the break from 15:00-15:15. It is recommended to visit exercise groups on a regular basis and to solve exercise sets independently.

The first exercise will be held on April 1 where the exercises on the back side of this sheet will be discussed. Exercise 1 below is due to April 7.

The exercise groups are

No.	**Time**	**Place**	**Assistant**
1	FR 8:15-10:00	HG E21	Robert Berke
2	FR 8:15-10:00	HG F26.3	Leo Rüst
3	FR 8:15-10:00	HG G26.5	Philipp Zumstein
4	FR 8:15-10:00	LFW C11	Dieter Mitsche
5	FR 10:15-12:00	HG F26.3	Birgitta Weber & Marc Nunkesser
6	FR 10:15-12:00	HG E1.2	Leon Peeters

Exercise sets, further informations about the lecture and a link to subscribe for exercise groups can be found on

http://www.ti.inf.ethz.ch/ew/courses/TI05/

Exercise 1

Exercises 1.4, 1.5, 1.7 and 1.14 in the hand-out *Random(ized) Search Trees.*

Due-date: 07.04.2005 in the lecture

The following exercises will be solved in the first exercise class on April 1, 2005.

Exercise 2

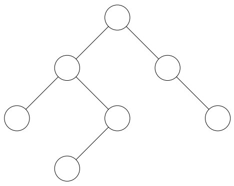
We choose $\{A, B\} \in_{u.a.r.} \binom{\{1...6\}}{2}$, and then set $C := A + B$.

(a) For $i \in \{1 \ldots 12\}$, determine $Pr[C = i]$.

(b) Determine $E[C]$ once by calculating and once by thinking.

Exercise 3

Compute the probability of the following random search tree generated by inserting elements uniformly at random.



Exercise 4

(a) Prove Markov's Inequality which is:

Let X be a nonnegative random variable (i.e. $X(\Omega) \subseteq \mathbf{R}_0^+$) for which $E[X] > 0$ exists. Then, for all $\lambda \in \mathbf{R}^+$,

$$Pr[X \geq \lambda E[X]] \leq \frac{1}{\lambda}.$$

Hint: Use that $E[X] = Pr[X < t] \cdot E[X|X < t] + Pr[X \geq t] \cdot E[X|X \geq t]$ for any $t \in \mathbf{R}^+$.

(b) Define a probability space and a random variable such that the Markov Inequality is fulfilled with

(i) identity, or

(ii) strict inequality.

Exercise 5

Determine closed forms for the following recursively defined series:

(a) For $n \in \mathbf{N}$,

$$a_n = \begin{cases} 1, & \text{if } n = 1, \text{ and} \\ 1 + \frac{1}{n}\sum_{i=1}^{n} a_i, & \text{otherwise.} \end{cases}$$

(b) For $n \in \mathbf{N}$,

$$b_n = \begin{cases} 1, & \text{if } n = 1, \text{ and} \\ 1 + \frac{n}{n-1} b_{n-1}, & \text{otherwise.} \end{cases}$$