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Theoretische Informatik (Kernfach) — *SS 2005*

Exercise Set 6

Exercise 1

Which of the following statements are true? Justify your answer by a counterexample, proof, reference to a theorem, etc.

(a) Every network has an integral flow.

(b) Every network has a maximum flow that is integral.

(c) Every network has a maximum flow.

(d) Every network with integral capacities has a unique maximum flow which, moreover, is integral.

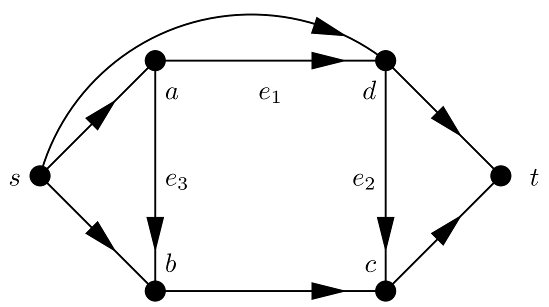
(e) A flow in a network is maximum if and only if its value is equal to the capacity of some s-t-cut.

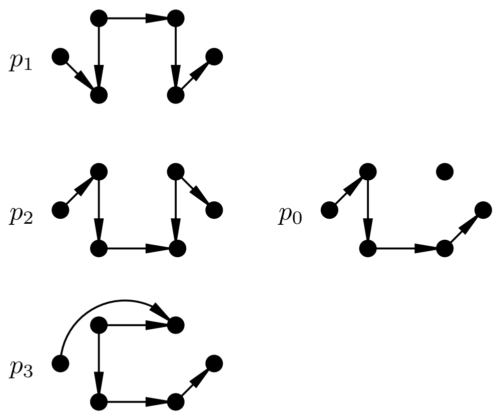
Exercise 2

The set of all flows on a given network is convex. Think this over, formulate it as a precise mathematical statement, and prove it.

Exercise 3

Consider the following network:





The edge-capacities of the network are:
$c(e_1) = 1, c(e_2) = \frac{\sqrt{5}-1}{2}, c(e_3) = 1$ and for all other edges e in G, $c(e) = M$, with $M \geq 4$.

(a) Simulate the Ford-Fulkerson algorithm on this network by augmenting with the paths p_0, p_1, p_2, p_1, p_3 (in this given order).

(b) Analyze the limit value of the flow if we augment with the following sequence of paths $p_0, p_1, p_2, p_1, p_3, p_1, p_2, p_1, p_3, p_1, p_2, p_1, p_3, \ldots$.

(c) Determine the maximum flow value of the given network.

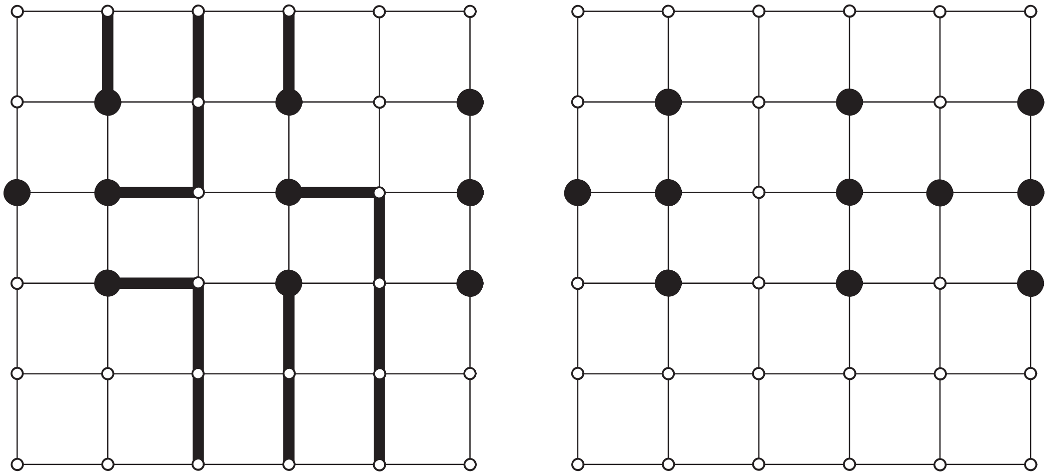


Figure 1: Two grids for the escape problem. Starting points are black, all other grid points white. In the left configuration an escape is possible whereas in the right one there is no escape.

Exercise 4

An $n \times n$ grid is an undirected graph consisting of n rows and n columns of vertices. (See Figure 1.) We denote the vertices in the ith row and the jth column by (i, j). All vertices in a grid have exactly four neighbors, except for the boundary vertices, which are the points (i, j) for which $i = 1$, $i = n$, $j = 1$, or $j = n$.

Given $m \leq n^2$ distinct starting points $(x_1, y_1), (x_2, y_2), \ldots, (x_m, y_m)$ in the grid, the escape problem is to determine whether or not there are m vertex-disjoint paths from the starting points to any m different points on the boundary. (See for example Figure 1.)

(a) Consider a flow network in which vertices, as well as edges, have capacities. That is, the positive net flow entering any given vertex is subject to a capacity constraint. Show that determining the maximum flow in a network with edge and vertex capacities can be reduced to an ordinary maximum flow problem on a flow network of comparable size.

(b) Describe an efficient algorithm to solve the escape problem.
(*Hint:* You might consider part (a).)

Due-date: 26.05.05 in the lecture