Eidgenössische Technische Hochschule Zürich
Ecole polytechnique fédérale de Zurich
Politecnico federale di Zurigo
Swiss Federal Institute of Technology Zurich

Institut für Theoretische Informatik
Emo Welzl, Jiří Matoušek

09.06.2005

Theoretische Informatik (Kernfach) *SS 2005*
Exercise Set 9

Exercise 1

Consider a modified checker for matrix multiplication (over the reals): Instead of choosing a random vector with components 0 and 1, choose a random vector with components drawn from $\{0, 1, \ldots, N-1\}$ uniformly and independently at random. Show that the probability of failure (declaring an incorrect multiplication correct) is at most $\frac{1}{N}$.

Exercise 2

Given a finite set S of rational numbers and positive integers d and n, $d \leq |S|$, find a polynomial $p(x_1, x_2, \ldots, x_n)$ of degree d for which the Schwartz–Zippel theorem is tight. That is, the number of n-tuples $(r_1, \ldots, r_n) \in S^n$ with $p(r_1, \ldots, r_n) = 0$ is $d|S|^{n-1}$.

Exercise 3

Suppose that we have an algorithm for testing the existence of a perfect matching in a given graph, with running time at most $T(n)$ for any n-vertex graph.

(a) Explain how repeated calls to the algorithm can be used to find a perfect matching if one exists. Estimate the running time of the resulting algorithm.

(b) How can the algorithm be used for finding a maximum matching in a given graph?

Exercise 4 (Challenge)

In the lecture notes (chapter 3.7, page 17) it is stated that the Dinits algorithm used for the maximum bipartite matching problem needs at most $O(m\sqrt{n})$ time. For the proof of this statement, one needs the following lemma. Prove it.

Lemma: After a phase with augmenting paths of length k, the value of the current flow differs from the maximum value by at most n/k.

Hint: Consider a decomposition of an integral maximum flow in the residual network into unit flows along paths (see Exercise 4 of Set 7).

Due-date: 16.06.05 in the lecture