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Approximation Algorithms and Semidefinite Programming FS12
Homework 2

Course Webpage: `http://www.ti.inf.ethz.ch/ew/lehre/ApproxSDP12/`
Due date: April 6, 2012, 23h59.

- Solutions are to be handed in typed in LaTeX. A tutorial can be found at `http://www.cadmo.ethz.ch/education/thesis/latex`.
- Note that there is no lecture on Friday, April 6 !
  - You may bring a print-out of your solution to my office (CAB G39.3) on Thursday, April 5, from 11h15 -12h.
  - Or you alternatively send your solution as a PDF to `stich@inf.ethz.ch` (no later than the deadline).
- Proofs are to be formally correct, complete and clearly explained. Be short and precise!

Exercise 1 (Sum of the k largest eigenvalues) *(10 Points)*

[Exercise 4.11] Let $C \in \mathrm{SYM}_n$.

a) Prove that the value of the following cone program is the sum of the k largest eigenvalues of C.

$$\begin{array}{ll} \min & ky + Tr(Y) \\ \text{s.t.} & yI_n + Y - C \succeq 0 \\ & (Y, y) \in \mathrm{PSD}_n \oplus \mathbb{R}\,. \end{array}$$

b) Derive the dual program (see Section 4.7) and show that its value is also the sum of the k largest eigenvalues of C.

Exercise 2 (A Semidefinite Program for the Theta Function) *(10 Points)*

[Exercise 4.12] For a given graph $G = (V, E)$ with $V = \{1, 2, \dots, n\}$, consider the semidefinite program

$$\begin{array}{ll} \text{maximize} & J_n \bullet X \\ \text{subject to} & \mathrm{Tr}(X) = 1 \\ & x_{ij} = 0,\ \{i, j\} \in E \\ & X \succeq 0\,, \end{array}$$

where J_n is the all-one $n \times n$ matrix, and show that its value is $\vartheta(G)$.

Exercise 3 (Theta Function of the strong product) *(5 Points)*

[Exercise 4.12] Prove that Lemma 3.4.2 actually holds with equality. You have to show

$$\vartheta(G \cdot H) \geq \vartheta(G)\vartheta(H)$$

for all graphs G, H.

Hint - Exercise 1.a) You may use the statement of Exercise 4.10.

Hint - Exercise 2) Dualize the semidefinite program in Theorem 3.6.1.

Hint - Exercise 3) Use the expression of $\vartheta(G)$ from Exercise 2.